Suurten poikkeamien teorian harj. 3, 28.10.-15

1. Koska $\Lambda^*(x) \geq \langle t_0, x\rangle - \Lambda(t_0) = \infty$, on $\Lambda^*(x) = \infty$, $\forall x \in \mathbb{R}^d$. Lauseen 3.1 nojalla

$$\overline{\lim}\; n^{-1} \log \mathbb{P}\left(\tfrac{S_n}{n} \in F\right) \leq -\Lambda^*(F), \;\forall F \text{ kompakti},$$

$$\underline{\lim}\; n^{-1} \log \mathbb{P}\left(\tfrac{S_n}{n} \in G\right) \geq -\Lambda^*(G) = -\infty, \;\forall G \text{ avoin}$$

triviaalisti. Siis HSPP, $I = \Lambda^*$.

2. Jos $x = (x_1, \dots, x_d)$, $y = (y_1, \dots, y_d)$, niin

$$\begin{cases} |x-y| < \delta \Rightarrow |x_i - y_i| < \delta, \forall i, \quad \text{ja} \\ |x_i - y_i| < \delta, \forall i \Rightarrow |x-y| < \delta\sqrt{d} \end{cases}$$

$$\overline{\lim}\; n^{-1} \log \mathbb{P}(\bar{X}_n \in B(x,\delta))$$

$$\leq \overline{\lim}\; n^{-1} \log \prod_{i=1}^{d} \mathbb{P}(\bar{X}_{n,i} \in B(x_i, \delta))$$

$$\leq -I_1(\bar{B}(x_1,\delta)) - \dots - I_d(\bar{B}(x_d,\delta)) \to -\sum_{i=1}^{d} I_i(x_i), \; \delta \to 0,$$

$$\underline{\lim}\; n^{-1} \log \mathbb{P}(\bar{X}_n \in B(x,\delta))$$

$$\geq \underline{\lim}\; n^{-1} \log \prod_{i=1}^{d} \mathbb{P}\left(\bar{X}_{n,i} \in B\left(x_i, \tfrac{\delta}{\sqrt{d}}\right)\right)$$

$$\geq -\sum_{i=1}^{d} I_i\left(B\left(x_i, \tfrac{\delta}{\sqrt{d}}\right)\right) \to -\sum_{i=1}^{d} I_i(x_i)$$

Edellisten havaintojen ja lauseen 2.1 nojalla HSPP väitetyllä vauhtifunktiolla.

3. a) $$g^*(x) = \sup_t \{ \langle t, x\rangle - f(t+a) \}$$

$$= -\langle a, x\rangle + \sup_t \{ \langle t+a, x\rangle - f(t+a) \}$$

$$= f^*(x) - \langle a, x\rangle.$$

b) $$h^*(x) = \sup_t \{ \langle t, x\rangle - b f(t) \}$$

$$= b \sup_t \{ \langle t, \tfrac{x}{b} \rangle - f(t) \} = b f^*(\tfrac{x}{b}).$$

4. $$\Lambda_n(t) = n^{-1} \log \mathbb{E}\left(e^{t \sum_{k=1}^{n} (a_1 \xi_k + \dots + a_N \xi_{k-N+1})} \right)$$

$$= n^{-1} \log \mathbb{E}\left(e^{ta \sum_{k=1}^{n-N+1} \xi_k} \right) \mathbb{E}\left(e^{b_1 \xi_{n-N+2}}\right) \cdots \mathbb{E}\left(e^{b_N \xi_N}\right),$$

missä $|b_i| \le ta$. Siispä

$$\Lambda(t) = \lim \Lambda_n(t) = \log(1 - p + p e^{at}) \quad \text{ja}$$

$$\Lambda^*(x) = \frac{x}{a} \log \frac{x}{a} + \left(1 - \frac{x}{a}\right) \log\left(1 - \frac{x}{a}\right) - \frac{x}{a} \log p + \left(\frac{x}{a} - 1\right) \log(1-p)$$

$(\Lambda^*(x) = \infty$, jos $x < 0$ tai $x > a)$.

5. a) $$\overline{\lim}\, n^{-1} \log P_n(|X_n| \ge B) \le -\inf_{|x| \ge B} I(x) \to -\infty, \; B \to \infty$$ (I hyvä)

$\Rightarrow \{P_n\}$ eksp. tiukka

b) Jos $\{P_n\}$ eksp. tiukka ja $\alpha > 0$, B:

$$\overline{\lim}\, n^{-1} \log \mathbb{P}(|X_n| \ge B) \le -\alpha,$$

niin

$$-\inf_{|x| > B} I(x) \le \underline{\lim}\, n^{-1} \log \mathbb{P}(|X_n| > B) \le -\alpha,$$

joten $\inf_{|x| > B} I(x) \ge \alpha$ ja $I(x) \to \infty$, kun $|x| \to \infty$

Näin ollen I on hyvä.