INTRODUCTION TO NUMBER THEORY. (Fall 2015)

5. EXERCISES (Mo 12.10)

1. Determine the quadratic residues (mod 19) by two ways: first directly from the definition, and the using Euler's criterion.

2. Let p be and odd prime. Show that

$$\sum_{j=0}^{p-1} \left(\frac{j}{p}\right) = 0.$$

3. Assume that p is an odd prime and fix an integer a that is a primitive root (mod p).

(i) Show that a^j is a quadratic residue if and only if j is even (and do this without using Thm 4.4 of lectures, which says that the number of quadratic residues is equal to the number of quadratic nonresidues).

(ii) Use (i) to Prove again Thm 4.4.

4. Find the number of solutions to the congruence $x^2 \equiv 761 \pmod{920}$.

5. Compute the Legendre symbols $\left(\frac{52}{97}\right)$ and $\left(\frac{240}{773}\right)$.

6. Determine $\left(\frac{5}{p}\right)$ for primes $p \geq 7$ using the quadratic reciprocity theorem.

7. Determine $\left(\frac{5}{p}\right)$ for primes $p \geq 5$ using the quadratic reciprocity theorem.

8*. Let X be the sum of the third powers of the quadratic residues (mod p), where p is an odd prime. Can you determine what is X conguent to (mod p) ?

Hints:

E.4: [Use theorem 4.3]

E.7: [After using the reciprocity Thm, consider different cases (mod 12).]