Homotopiateoria
Harjoitus 2 (17.9.2015)

1. a) Väisälä, s. 154, tehtävä 21:6.

b) Osoita, että avaruus on kutistuva jos ja vain jos se on homotopiaekvivalentti yksiön kanssa.

2. Väisälä, s. 154, tehtävä 21:5

3. Väisälä, s. 159, tehtävä 22:2

4. Väisälä, s. 159, tehtävä 22:4

5. Väisälä, s. 159, tehtävä 22:5

6. Väisälä, s. 154, tehtävä 21:8

Homotopy theory
Exercise 2 (17.9.2015)

1. a) Väisälä, p. 154, Exercise 21:6

Suppose that $f, g\colon X \to Y$ are continuous maps and Y is contractible. Prove that $f \simeq g$.

b) Prove that a space is contractible if and only if it is homotopy equivalent with a singleton set (a set with one element).

2. Väisälä, p. 154, Exercise 21:5

Let $X \subset \mathbb{R}^2$ be the union of the coordinate axes and the line segment $x + y = 1$, $0 \le x \le 1$. Prove that $X \simeq S^1$.

3. Väisälä, p. 159, Exercise 22:2

Prove: If X is path connected, then any two paths $\alpha, \beta\colon I \to X$ are homotopic with each other (when we don't require the condition rel $\{0, 1\}$).

4. Väisälä, p. 159, Exercise 22:4

Let $\alpha\colon I \to X$ be a path and denote $b = \alpha(1)$. Write down an explicit formula for a path homotopy $H\colon \alpha^{\leftarrow}\alpha \sim \epsilon_b$.

5. Väisälä, p. 159, Exercise 22:5

Let X be a path connected space and $z \in X$. Prove that any path in X is path homotopic with some path which goes through z (that is, has the value z).

6. Väisälä, p. 154, Exercise 21:8

Let $X \subset \mathbb{R}^2$ be the so called *comb space*

$$X = I \times \{0\} \cup \{0\} \times I \cup \bigcup_{j \in \mathbb{N}}(\{1/j\} \times I).$$

Prove that the space X is contractible, but the pointed space $(X, (0, 1))$ is not base point contractible.