Homotopy theory
Exercise 10 (19.11.2015)

1. Prove Proposition 6.17.

2. Prove the formula

$$h_{\bar{\alpha}}(\bar{\beta}) = \bar{\alpha}\bar{\beta}\overline{\alpha^{\leftarrow}}$$

in Example 6.19.

3. See Remark 6.21.

a) Prove: If X is path connected, and $\pi_1(X, x_0)$ acts trivially in the group $\pi_n(X, x_0)$ for some $x_0 \in X$, then $\pi_1(X, x)$ acts trivially in the group $\pi_n(X, x)$ for every $x \in X$.

b) Prove: A path connected space is 1-simple if and only if $\pi_1(X, x_0)$ is an Abelian group.

4. Prove Proposition 6.23.

5. Consider the lifting problem

$$\begin{tikzcd}
 & \mathbb{R} \times \mathbb{R} \arrow[d, "pr_1"] \\
I \arrow[ur, dotted, "\hat{i}"] \arrow[r, hook, "i"'] & \mathbb{R}
\end{tikzcd}$$

"Make a list" of all lifts $\hat{i}$, for which $\hat{i}(0) = (0, 0)$. Compare with the situation where instead of $\mathbb{R} \times \mathbb{R}$ have $\mathbb{R} \times \mathbb{Z}$.

6. Väisälä, p. 175, Exercise 24:4.

Suppose that $p\colon X \to Y$ is a covering map and Y is path connected. Prove that every fiber $p^{-1}\{y\}$ has the same cardinality, that is, for every $y_1, y_2 \in Y$ there exists a bijection $p^{-1}\{y_1\} \to p^{-1}\{y_2\}$. [Hint: path lifting]

Homotopiateoria
Harjoitus 10 (19.11.2015)

1. Todista Lause 6.17.

2. Todista Esimerkin 6.19 kaava

$$h_{\bar{\alpha}}(\bar{\beta}) = \bar{\alpha}\bar{\beta}\overline{\alpha^{\leftarrow}}.$$

3. Kts. Huomautus 6.21.

a) Osoita: Jos X on polkuyhtenäinen ja $\pi_1(X, x_0)$ toimii triviaalisti ryhmässä $\pi_n(X, x_0)$ jollakin $x_0 \in X$, niin $\pi_1(X, x)$ toimii triviaalisti ryhmässä $\pi_n(X, x)$ jokaisella $x \in X$.

b) Osoita: Polkuyhtenäinen avaruus on 1-yksinkertainen jos ja vain jos $\pi_1(X, x_0)$ on Abelin ryhmä.

4. Todista Lause 6.23.

5. Tarkastellaan nosto-ongelmaa

$$\begin{tikzcd} & \mathbb{R} \times \mathbb{R} \arrow[d, "pr_1"] \\ I \arrow[ur, dotted, "\hat{\imath}"] \arrow[r, hook, "i"'] & \mathbb{R} \end{tikzcd}$$

"Luettele" tavalla tai toisella kaikki nostot $\hat{\imath}$, joille $\hat{\imath}(0) = (0, 0)$. Vertaa tilanteeseen, jossa $(\mathbb{R} \times \mathbb{R})$:n tilalla on $\mathbb{R} \times \mathbb{Z}$.

6. Väisälä, s. 175, tehtävä 24:4.