Euler Γ and B functions, Riemann ζ-function and their properties:

$$\Gamma(z) = \int_0^\infty e^{-t} t^{z-1} dt \qquad \Gamma(n+1) = n! \qquad \Gamma(z+1) = z\Gamma(z) \qquad \Gamma(z)\Gamma(1-z) = \frac{\pi}{\sin \pi z}$$

$$\zeta(z) = \sum_{k=1}^\infty \frac{1}{k^z} \qquad B(p,q) = \int_0^1 t^{p-1}(1-t)^{q-1} dt \qquad B(p,q) = \frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)}$$

In all of the inverse formulae below, if x is a point of discontinuity of f one needs to replace "$f(x)$" by "$(f(x^+)+f(x^-))/2$".

Fourier series:

If f is L-periodic:

$$f(x) = \sum_{k=-\infty}^{\infty} c_k e^{i2\pi kx/L} \qquad c_k = \frac{1}{L}\int_0^L f(x) e^{-i2\pi kx/L} dx$$

If f is 2π-periodic:

$$f(x) = \sum_{k=-\infty}^{\infty} c_k e^{ikx} \qquad c_k = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x) e^{-ikx} dx$$

$$f(x) = \frac{a_0}{2} + \sum_{k=1}^\infty (a_k \cos(kx) + b_k \sin(kx)) \qquad a_k = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos(kx)dx \qquad b_k = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin(kx)dx$$

Fourier transform, inverse transform, and basic properties (as defined in Arfken & Weber):

$$\mathcal{F}[f] = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} dx\, f(x) e^{-ikx} \qquad f(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} dk\, \mathcal{F}[f] e^{ikx} \qquad f * g = \int_{-\infty}^{\infty} f(x-y)g(y)dy$$

$$\mathcal{F}_s[f] = \sqrt{\frac{2}{\pi}}\int_0^\infty f(x)\sin(kx)dx \qquad f(x) = \sqrt{\frac{2}{\pi}}\int_0^\infty \mathcal{F}_s[f]\sin(kx)dk \qquad \mathcal{F}[f*g] = \sqrt{2\pi}\mathcal{F}[f]\mathcal{F}[g]$$

$$\mathcal{F}_c[f] = \sqrt{\frac{2}{\pi}}\int_0^\infty f(x)\cos(kx)dx \qquad f(x) = \sqrt{\frac{2}{\pi}}\int_0^\infty \mathcal{F}_c[f]\cos(kx)dk \qquad \mathcal{F}[f'] = ik\mathcal{F}[f]$$

$$\mathcal{F}[f](\mathbf{k}) = \frac{1}{(2\pi)^{d/2}}\int_{\mathbb{R}^d} d\mathbf{x} f(\mathbf{x}) e^{-i\mathbf{k}\cdot\mathbf{x}} \qquad f(\mathbf{x}) = \frac{1}{(2\pi)^{d/2}}\int_{\mathbb{R}^d} d\mathbf{k}\mathcal{F}[f] e^{i\mathbf{k}\cdot\mathbf{x}}$$

Parseval formulae for Fourier series and transform:

$$\int_{-\pi}^{\pi} |f(x)|^2 dx = 2\pi \sum_{k=-\infty}^{\infty} |c_k|^2 = \pi\left[\frac{|a_0|^2}{2} + \sum_{k=1}^\infty (|a_k|^2 + |b_k|^2)\right] \qquad \int_{-\infty}^{\infty} |f(x)|^2 dx = \int_{-\infty}^{\infty} |\mathcal{F}[f]|^2 dk$$

Laplace transform, inverse transform, and basic properties:

$$\mathcal{L}[f] = \int_0^\infty e^{-st} f(t) dt \qquad f = \int_{\sigma-i\infty}^{\sigma+i\infty} \mathcal{L}[f] e^{st} \frac{ds}{2\pi i} \qquad \mathcal{L}[af+bg] = a\mathcal{L}[f] + b\mathcal{L}[g]$$

$$\mathcal{L}[-tf] = \frac{d\mathcal{L}[f]}{ds} \qquad \mathcal{L}[f'] = s\mathcal{L}[f] - f(0^+) \qquad \mathcal{L}\left[\int_0^t f(\tau)g(t-\tau)d\tau\right] = \mathcal{L}[f]\mathcal{L}[g]$$