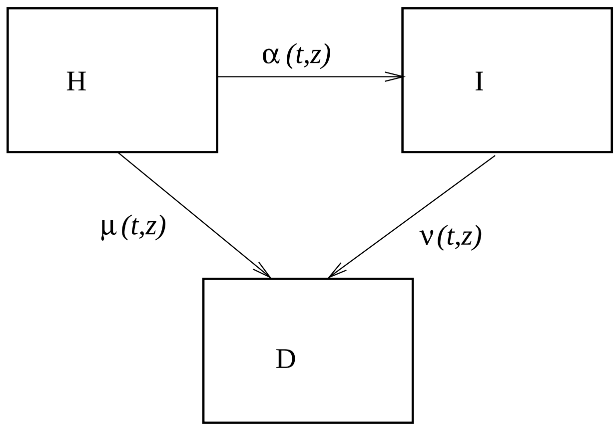
Health-illness model revisited



In general, rates $\alpha(t, z)$, $\mu(t, z)$ and $\nu(t, z)$ may depend on calendar time t and age z.

Open cohort

- Consider an *open* cohort, in which new individuals are born in calendar time with rate $\beta(t)$ (cf. the Lexis diagram)
- How does the population look with respect to the healthy-ill status of individuals at a given time point t?
  - What is the proportion of ill individuals?
  - What is the proportion of ill of all those of age z?
  - What is the relation between incidence and prevalence?
- We'll consider the *stationary* situation in particular

Keiding N: Age-specific incidence and prevalence (JRSS A, 1991)

Healthy individuals of age z at time t

- The number of healthy individuals of age $[z, z+dz[$ at time t:

$$\beta(t-z)k(t,z)dz \equiv$$

$$\beta(t-z)\exp\left(-\int_0^z(\mu(t-z+u,u)+\alpha(t-z+u,u)du\right)dz$$

Ill invidividuals of age z at time t

- The number of ill individuals of age $[z, z+dz[$ at time t, with onset of diseat at age $[y, y+dy[$

$$\beta(t-z)h(t,y,z)dydz \equiv$$

$$\beta(t-z)\exp\left(-\int_0^y(\mu(t-z+u,u)+\alpha(t-z+u,u))du\right)\times$$

$$\alpha(t-z+y,y)\times\exp(-\int_y^z\nu(t-z+u,u)du)dz$$

- The number of ill individuals of age $[z, z+dz[$ at time t:

$$\beta(t-z)\int_0^z h(t,y,z)dydz$$

Stationarity

- Assume stationarity in the sense that the rates do not depend on calendar time
- It then follows that at any time t the numbers of healthy and ill of age $[z, z+dz[$ are:

$$\beta k(z)dz = \beta \exp(-\int_0^z (\mu(u) + \alpha(u))du)dz \text{ (healthy)}$$

$$\beta \int_0^z h(y,z)dydz = \beta \int_0^z k(y)\alpha(y) \exp(-\int_y^z \nu(u)du)dydz \text{ (ill)}$$

Distribution of healthy-ill status

- Under stationarity the probability that an individual is healthy and of age $[z, z+dz[$ thus is

$$\frac{k(z)dz}{C}$$

- Likewise, the probability that an individual is ill and of age $[z, z+dz[$ is

$$\frac{\int_0^z h(y,z)dydz}{C}$$

- The normalising constant C is the mean life length (see next slide)

Mean times as healthy and ill

- The normalising constant

$C = \int_0^\infty k(z)dz + \int_0^\infty \int_0^z h(y,z)dydz = M_H + M_I$

where

M_H = the mean time spent healthy

M_I = the mean time spent ill (see next slide)

Mean time spent ill cont.

- The mean time spent ill equals the risk R of ever falling ill times the mean duration D of illness (in those that fall ill):

$M_I = \int_0^\infty \int_0^z h(y,z)dydz = \int_0^\infty \int_y^\infty h(y,z)dzdy =$

$\int_0^\infty \int_y^\infty k(y)\alpha(y)\exp(-\int_y^z \nu(u)du)dzdy =$

$\int_0^\infty k(y)\alpha(y)\mathsf{E}(X - Y|Y = y)dy =$

$R \times \mathsf{E}(X - Y|$the individual falls even ill$) \equiv R \times D$

- Here X is the time of death and Y is the time of falling ill

Age-specific prevalence of ill individuals

- Conditioning on age, we can now write an expression for the proportion of ill individuals of all those of age z:

$$\frac{\int_0^z h(y,z)dy}{k(z) + \int_0^z h(y,z)dy}$$

Prevalence of ill individuals

- Marginalising, we can write the proportion of ill inividuals in the population as:

$$\frac{\int_0^\infty \int_0^z h(y,z)dydz}{\int_0^\infty k(z)dz + \int_0^\infty \int_y^z h(y,z)dydz} = \frac{M_I}{M_H + M_I}$$

Prevalence and incidence numbers

- The prevalenve number of ill individuals

$\beta \int_0^\infty \int_y^\infty h(y,z) dz dy = (\beta R) \times D$

- The incidence (number) of new cases per time unit:

$\beta \int_0^\infty k(y) \alpha(y) dy = \beta R$

- It follows that “prevalence number = incidence number x mean duration"

Prevalence, incidence and mean duratio

- Based on the prevalence of ill individuals (see above),

Prevalence odds of ill individuals $= \frac{M_I}{M_H} = \frac{R}{M_H} \times D$

- If rate α does not depend on age, $R/M_H = \alpha$, so "prevalence odds = incidence x mean duration"