VEKTORIANALYYSI / CALCULUS OF SEVERAL VARIABLES
LASKUHARJOITUS 1 / EXERCISE 1
SYKSY 2014 / AUTUMN 2014

1. Laske sisätulo $\bar{x} \cdot \bar{y}$, kun
a) $\bar{x} = (-2, 0, 4)$ ja $\bar{y} = (10, 5, 5)$
b) $\bar{x} = (-1, -1, 1, 1)$ ja $\bar{y} = (2, 3, 0, 2)$
Ovatko vektorit $\bar{x}$ ja $\bar{y}$ kohtisuorassa toisiaan vastaan?

2. Piirrä $\mathbb{R}^3$:n koordinaatistoon joukot
a) $A := B((-2, 0, 0), 2)$,
b) $B := \Big\{(x, y, z) \mid 0 \le x \le 2,\ 1 \le y \le 4,\ 1 \le z \le 4, \Big\}$,
c) joukko $A \cap B_1$, kun
$B_1 := \Big\{(x, y, z) \mid y = 0\Big\}$

3. Muodosta seuraavien vektorijonojen $(\bar{y}^{(k)})_{k=1}^{\infty}$ komponenttijonot. Suppenevatko ko. vektorijonot? Jos suppenevat, laske raja-arvot.

$$\text{a)}\quad \bar{y}^{(k)} = \Big(\frac{5+k}{k^2}, \frac{(-1)^k}{k}, \frac{2+k}{k}, \frac{1}{k^2}\Big) \in \mathbb{R}^4$$
$$\text{b)}\quad \bar{y}^{(k)} = \Big(2,\ \frac{\sin(k\pi/4)}{k},\ \sin(k\pi/4)\Big) \in \mathbb{R}^3$$

4. Tarkastellaan seuraavia tason osajoukkoja:
$$A = \{(x, y) \in \mathbb{R}^2 \,|\, x > y - 2\}\ ,\ B = \{(x, y) \in \mathbb{R}^2 \,|\, x > y - 2, y \le 0\},$$
$$C = \{(x, y) \in \mathbb{R}^2 \,|\, y = 2\}$$
Ovatko nämä avoimia tai suljettuja?

5. Onko funktiolla $f : \mathbb{R}^2 \setminus \{\bar{0}\} \to \mathbb{R}$,
$$f(x, y) = \frac{x^4 - xy^3}{x^4 + y^2}\ ,$$
raja-arvoa origossa?

6. Olkoon $\bar{x} = (x_1, \dots, x_n) \in \mathbb{R}^n$, $n = 2, 3, \dots$. Käytämme vektorin pituudelle merkintää $|\bar{x}| = \big(\sum_{j=1}^{n} |x_j|^2\big)^{1/2}$, ja lisäksi merkitään
$$\|\bar{x}\|_\infty = \max_{j=1,\dots,n} |x_j| \quad \text{ja} \quad \|\bar{x}\|_1 = \sum_{j=1}^{n} |x_j|.$$
Osoita, että
$$\frac{1}{\sqrt{n}}|\bar{x}| \le \|\bar{x}\|_\infty \le |\bar{x}| \le \|\bar{x}\|_1 \le \sqrt{n}|\bar{x}|.$$
(Cauchy-Schwartzin epäyhtälöstä voi olla apua.)
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1. Calculate the scalar product $\bar{x} \cdot \bar{y}$ for

a) $\bar{x} = (-2, 0, 4)$ ja $\bar{y} = (10, 5, 5)$
b) $\bar{x} = (-1, -1, 1, 1)$ ja $\bar{y} = (2, 3, 0, 2)$
Are the vectors $\bar{x}$ and $\bar{y}$ orthogonal to each other?

2. Draw figures presenting the following subsets of $\mathbb{R}^3$:
a) $A := B((-2, 0, 0), 2)$,
b) $B := \Big\{(x, y, z) \mid 0 \leq x \leq 2,\ 1 \leq y \leq 4,\ 1 \leq z \leq 4, \Big\}$,
c) the set $A \cap B_1$ with
$B_1 := \Big\{(x, y, z) \mid y = 0\Big\}$

3. Write the coordinate (or component) sequences for the following vector sequences $(\bar{y}^{(k)})_{k=1}^{\infty}$. Do the vector sequences converge, and if so, then calculate the limits.

$$\text{a)}\ \ \bar{y}^{(k)} = \Big(\frac{5+k}{k^2}, \frac{(-1)^k}{k}, \frac{2+k}{k}, \frac{1}{k^2}\Big) \in \mathbb{R}^4$$
$$\text{b)}\ \ \bar{y}^{(k)} = \Big(2,\ \frac{\sin(k\pi/4)}{k},\ \sin(k\pi/4)\Big) \in \mathbb{R}^3$$

4. Consider the following subsets of the plane:

$$A = \{(x, y) \in \mathbb{R}^2 \mid x > y - 2\}\ ,\ B = \{(x, y) \in \mathbb{R}^2 \mid x > y - 2, y \leq 0\},$$
$$C = \{(x, y) \in \mathbb{R}^2 \mid y = 2\}$$

Are they open or closed?

5. Does the function $f : \mathbb{R}^2 \setminus \{\bar{0}\} \to \mathbb{R}$,

$$f(x, y) = \frac{x^4 - xy^3}{x^4 + y^2}\ ,$$

have a limit at the origin?

6. Let $\bar{x} = (x_1, \ldots, x_n) \in \mathbb{R}^n$, $n = 2, 3, \ldots$, and denote its length by $|\bar{x}| = \big(\sum_{j=1}^{n} |x_j|^2\big)^{1/2}$. In addition we write

$$\|\bar{x}\|_\infty = \max_{j=1,\ldots,n} |x_j| \quad \text{ja} \quad \|\bar{x}\|_1 = \sum_{j=1}^{n} |x_j|.$$

Show that

$$\frac{1}{\sqrt{n}}|\bar{x}| \leq \|\bar{x}\|_\infty \leq |\bar{x}| \leq \|\bar{x}\|_1 \leq \sqrt{n}|\bar{x}|.$$

(The Cauchy-Schwartzin inequality may be of some help.)