VEKTORIANALYYSI / CALCULUS OF SEVERAL VARIABLES
LASKUHARJOITUS / EXERCISE 12
SYKSY / AUTUMN 2014

1. Olkoon

$$G := \{x = (x_1, x_2, x_3) \in \mathbb{R}^3 \mid 0 < x_3 < 2, x_1^2 + x_2^2 < 4\}.$$

Määritä vektorikentän $F(x_1, x_2, x_3) := (0, 0, x_3)$ vuo ulospäin pinnan ∂G läpi.

2. Laske vektorikentän a) $F(x_1, x_2, x_3) := (e^{x_1}, x_3 - x_2, x_1)$,
b)

$$H(x_1, x_2, x_3) := \frac{x}{1 + |x|^2}$$

divergenssi, kun $x = (x_1, x_2, x_3)$.
c) Laske vektorikentän $F(x_1, x_2, x_3) := (2x_3, -4x_1^2, \arctan x_1)$ roottori.

3. Laske vektorikentän $F(x, y, z) := (x^2 + y^2, y^2 - z^2, z)$ vuo pallopinnan

$$S := \{(x, y, z) \mid x^2 + y^2 + z^2 = a^2\}$$

läpi, kun $a > 0$ on vakio.

4. Kuten tehtävä 3, mutta
a) $F(x, y, z) := (ye^z, x^2e^z, xy)$.
b) $F(x, y, z) := (x^3, 3yz^2, 3y^2z + x^2)$

5. Laske Stokesin lauseen avulla integraali

$$\int_\gamma F \cdot d\bar{s},$$

kun $F(x_1, x_2, x_3) := (2x_2, 3x_3, x_1)$ ja γ on kolmion reuna ja kolmion kärkipisteet ovat $(0, 0, 0)$, $(0, 2, 0)$ ja $(1, 1, 1)$.

6. Laske Stokesin lauseen avulla

$$\int_\gamma y dx - x dy + z^2 dz,$$

kun γ on sylinterien $z = y^2$ ja $x^2 + y^2 = 4$ leikkaus suunnistettuna vastapäivään, kun katsotaan kaukaa positiivisen z–akselin puolelta.

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1. Let

$$G := \{x = (x_1, x_2, x_3) \in \mathbb{R}^3 \mid 0 < x_3 < 2, x_1^2 + x_2^2 < 4\}.$$

Calculate the outward flux of the vector field $F(x_1, x_2, x_3) := (0, 0, x_3)$ through the surface ∂G.

2. Calculate the divergence of the vector field a) $F(x_1, x_2, x_3) := (e^{x_1}, x_3 - x_2, x_1)$,
b)

$$H(x_1, x_2, x_3) := \frac{x}{1 + |x|^2}$$

where $x = (x_1, x_2, x_3)$.
c) Calculate the curl of the vector field $F(x_1, x_2, x_3) := (2x_3, -4x_1^2, \arctan x_1)$.

3. Calculate the flux of the vector field $F(x, y, z) := (x^2 + y^2, y^2 - z^2, z)$ through the surface

$$S := \{(x, y, z) \mid x^2 + y^2 + z^2 = a^2\},$$

when $a > 0$ is a constant.

4. As problem 3 with
a) $F(x, y, z) := (ye^z, x^2e^z, xy)$.
b) $F(x, y, z) := (x^3, 3yz^2, 3y^2z + x^2)$

5. Use the Stokes theorem to determine the integral

$$\int_\gamma F \cdot d\bar{s},$$

when $F(x_1, x_2, x_3) := (2x_2, 3x_3, x_1)$ and γ is the boundary of a triangle with vertices at $(0, 0, 0)$, $(0, 2, 0)$ $(1, 1, 1)$.

6. Use the Stokes theorem to determine the integral

$$\int_\gamma y dx - x dy + z^2 dz,$$

when γ is the intersection of the cylinders $z = y^2$ and $x^2 + y^2 = 4$ with counterclockwise orientation, when seen from a distant point of the positive z-axis.