Partial Differential Equations II

HW 1

Return to Jussi by Monday, September 22nd

1. Let $u(x) = |x|^{1/2}$, $|x| \leq 1$. For which γ, $0 < \gamma \leq 1$, do we have $u \in C^{0,\gamma}((-1,1))$?

2. Show, that $C^{0,\gamma}(U)$ is a Banach-space with respect to norm $\|\cdot\|_{C^{0,\gamma}(U)}$. Here, as usual, $0 < \gamma \leq 1$, and $U \subset \mathbb{R}^n$ is open.

3. Prove in detail that a weak derivative is unique.

4. Assmue $0 < \beta < \gamma \leq 1$. Prove the so called *interpolation inequality*

$$\|u\|_{C^{0,\gamma}(U)} \leq \|u\|_{C^{0,\beta}(U)}^{\frac{1-\gamma}{1-\beta}} \|u\|_{C^{0,1}(U)}^{\frac{\gamma-\beta}{1-\beta}}.$$

   Before trying the next questions, read pages 240–260 in Evans. Do not worry too much about all the details in the proofs, we will worry about those in the lectures. However, try to form a general picture of the material covered.

5. Explain why Theorem 2 of subsection 5.3.1 is more useful than Theorem 1.

6. What does *straightening the boundary* mean? Starting from the definition of a C^1–domain, try to describe in detail why this works.