17 November 2014

Operator Algebras-A Tool Kit

Exercise Sheet 10

Department of Mathematics and Statistics
University of Helsinki, Fall 2014

Exercise 1: Show that a compact space X is connected if and only if the constant functions 0 and 1 are the only projections in $C(X)$.

Exercise 2: Show that the Bott element is actually a projection in $M_2(C(\mathbb{S}^2))$.

Exercise 3: Prove Proposition 6.1.11 and Proposition 6.1.12.

Exercise 4:

(a) Show that the Grothedieck group of the abelian semigroup $(\mathbb{N}_0, +, 0)$ is isomorphic to the group of integers $\mathbb{Z}$.

(b) Show that the Grothendieck group of the abelian semigroup $(\mathbb{Z} \setminus \{0\}, \cdot, 1)$, i.e., of the non-zero integers endowed with the obvious multiplication and identity 1 is isomorphic to $\mathbb{Q} \setminus \{0\}$.

(c) Compute the Grothendieck group of the abelian semigroup $(\mathbb{Z}, \cdot, 1)$, i.e., of the integers endowed with the obvious multiplication and identity 1.

Exercise 5: Show that if A is a unital C^*-algebra, then its unitalization $\widetilde{A}$ is isomorphic to $A \oplus \mathbb{C}$.

Exercise 6: Show that if p, q are projections in $M_n(\mathbb{C})$, then

$$p \sim q \Longleftrightarrow \text{range}(p) = \text{range}(q).$$

Conclude that $K_0(\mathbb{C}) \cong \mathbb{Z}$.