

Department of Mathematics and Statistics
Minimal Surfaces
Exercise 6
31.10.2014

Return by **Thursday, October 30**. This time we have only 4 exercises.

1. Let $D = \mathbb{C} \setminus \{z = x + iy : x \geq 0\}$ and let $f, g: D \rightarrow \mathbb{C}$ be analytic functions

$$f(z) = \frac{1}{z^2}, \quad g(z) = z$$

in the Weierstrass-Enneper representation. Try to find out which minimal surface (or a piece of a surface) you will obtain. [Hints: Choose e.g. $z_0 = i$ as a starting point in the integration from z_0 to z . It is helpful to use polar coordinates $z = re^{i\vartheta}$ with $r = e^t$ to express the surface in parameters t, ϑ .]

2. Let $g(z) = e^{iz}$ and $f(z) = e^{-iz}$ in the Weierstrass-Enneper representation. Find out which minimal surface you will obtain.
3. Compute the total curvature of the catenoid.
4. Let $F: \mathbb{R}^n \rightarrow \mathbb{R}$ be convex and C^1 -smooth. Prove that

$$F(x) - F(y) \geq \nabla F(y) \cdot (x - y)$$

for $x, y \in \mathbb{R}^n$.