Department of Mathematics and Statistics
Minimal Surfaces
Exercise 1
19.9.2014

Return by **Thursday, September 18**.

1. Prove that the area functional
$$\mathcal{F}(u) = \int_\Omega \sqrt{1 + |\nabla u|^2}$$
on graphs is strictly convex.

2. Prove that the minimal graph equation
$$\operatorname{div}\left(\frac{\nabla u}{\sqrt{1 + |\nabla u|^2}}\right) = 0$$
is equivalent to the equation
$$(1 + u_x^2)u_{yy} - 2u_x u_y u_{xy} + (1 + u_y^2)u_{xx} = 0.$$

3. Verify that we may assume the function u to be radial in Bernstein's example.

4. Prove that
$$u''(r) = -\frac{1}{r}\left(u'(r) + u'(r)^3\right)$$
is the Euler-Lagrange equation for the radial area functional
$$2\pi \int_\rho^R r\sqrt{1 + u'(r)^2}$$

5. Prove that Enneper's surface is obtained by choosing $f(z) = 1$ and $g(z) = z$ in the Weierstrass-Enneper parametrization. You will not get exactly the same parametrization as in lecture notes but don't worry. Explain the difference.

6. Prove that the standard connection $\bar{\nabla}$ on $\mathbb{R}^m$ is torsion-free and compatible with the standard inner product of $\mathbb{R}^m$.