University of Helsinki / Department of Mathematics and Statistics
SCIENTIFIC COMPUTING
Exercise 01, 8.9.2014

`Problem sessions will be held on Monday at 16-18`

N.B. The files mentioned in the exercises (if any) are available on the course homepage

1. Apply the recursion formula $x_0 = 1, x_{n+1} = \frac{1}{2}(x_n + \frac{a}{x_n}), n = 0, 1, 2, \ldots$ for $\sqrt{a}$ to compute $\sqrt{3}$. Print the results in the following format:

```
n  x(n)   Error
0  1
.....
6  ...
```

2. Approximations to the number π are given by the formula

$$p(n) = \sum_{k=0}^{n} \frac{1}{16^k} \left(\frac{4}{8k+1} - \frac{2}{8k+4} - \frac{1}{8k+5} - \frac{1}{8k+6} \right).$$

Print the first few results in the same format as in problem 1.

3. According to an Internet page, the center w of a circle through three points a, b, c in the complex plane can be found as follows in MATLAB notation:

```
u=(b-c).*abs(a).^2 + (c-a).*abs(b).^2 + (a-b).*abs(c).^2;
v=(b-c).*conj(a)+  (c-a).*conj(b)+  (a-b).*conj(c);
w= u./v;
```

Write a MATLAB script to check this claim. (Hint: Take three random points on the unit circle, then compute w and show that it is 0.)

4. Let $(x_j, y_j), j = 0, 1, ..., n$ be the vertices of a polygon with $(x_0, y_0) = (x_n, y_n)$. The area of the polygon is given by $a = \frac{1}{2}\sum_{i=1}^{n} t_i$ with $t_i = x_{i-1}y_i - x_i y_{i-1}$. Carry out the following steps for each of the regular polygons triangle, square and hexagon:

FILE: ~/MME/demo13/d01/d01.tex — 21. elokuuta 2014 (klo 11.23).

(a) Choose vertices and compute the area by school geometry.
(b) Compute the area by the formula and compare to the exact value.
(c) Plot the figure.

5. Hilbert's inequality says that for $a_k, b_k \geq 0$

$$\sum_{m=0}^{\infty}\sum_{n=0}^{\infty} \frac{a_m b_n}{m+n+1} \leq \pi(\sum_{m=0}^{\infty} a_m^2)^{1/2}(\sum_{n=0}^{\infty} b_n^2)^{1/2}.$$

Carry out a numerical verification of this inequality.

6. What does the following program do? Execute it and interprete the results.

```
% FILE d016.m begins.
for pp=1:3
a=2*rand; b=3*(a+1);
f=@(x)(a*sin(b*x)); v=quad(f,0,1);
exact=(a/b)*(1-cos(b));
fprintf(' %6.4f  %6.4f  %12.6f   %12.4e\n', a, b, v, v-exact)
end
% FILE d016.m ends.
```