COURSE JOURNAL /RHS

4.9.2014

Welcome to the course.
$(\mathbb{R}^2, +.\cdot)$ is a field.
$(\mathbb{R}^2, +.\cdot) =: \mathbb{C}$ is called the complex plane.
Elements of the complex plane are called complex numbers.
$z = x + iy$, $x, y \in \mathbb{R}$.
Definitions of conjugate of z, modulus of z, argument of z.
The triangle inequality.

5.9.2014

Argument-modulus-expression for z.
Geometric meaning of the multiplication of two complex numbers.
De Moivre formulas.

11.9.2014

Applications of De Moivre formulas.
Topology of the complex plane: for example, open set, closed set, boundary of a set, the interior of a set.

12.9.2014

Topology of the complex plane continued.
Limit of a function.
Continuity of a function.
Definition of an analytic function.
Examples of analytic functions: $f(z) = z^2$, $g(z) = z$, $h(z) = c$. Example of a function which is not analytic, $f : \mathbb{C} \to \mathbb{C}$, $f(z) = \overline{z}$.
The characterization formula for a function which has a complex derivative at a point z_0.
A function which has a complex derivative at z_0 is continuous at z_0.

Date: December 5, 2014.
Key words and phrases. Complex numbers, Complex functions, Analytic functions.

18.9.2014

Rules of complex derivatives of $f+g$, fg, f/g, and $g \circ f$.
Corollaries of the rules.
Examples where the rules can be used.
If f has a complex derivative at z_0, we proved that the Cauchy-Riemann equations are valid at z_0.

19.9.2014

We formulated and proved the following theorem which is called Cauchy-Riemann characterization theorem: Let $f : A \to \mathbb{C}$ be a function, $A \subset \mathbb{C}$ be open, and $z_0 \in A$. Then, f has a complex derivative at z_0, if and only if f is differentiable in the real sense at z_0 (where $f : \mathbb{R}^2 \to \mathbb{R}^2$ in natural way) and the Cauchy-Riemann equations are valid at z_0.
Examples of this theorem.

25.9.2014

On constant valued functions.
On harmonic functions.
A couple of examples.
We started the chapter of complex series.

26.9.2014

Abel's theorem.
Corollaries of Abel's theorem.
Cauchy-Hadamard formula.
Examples.

2.10.2014

A power series in its disk of convergence represents an analytic function.
Basic properties of the exponent function.

9.10.2014

Trigonometric functions.
Hyperbolic trigonometric functions.
The logarithm.

10.10.2014

Definition of a branch of the logarithm function.
Definition a branch of z^a.
Examples.

16.10.2014

Paths, smooth paths.
Integration along smooth paths.
Examples.

17.10.2014

Contour integration.
Technical lemmas of integrals along smooth paths: reparametrization of a path, integral along a join of paths, reversal.
The Estimation Lemma.
Examples.

30.10.2014

The Fundamental Theorem of Calculus.
Consequences of the Fundamental Theorem.
Examples.

31.10.2014

The Cauchy-Goursat Theorem in an open disk.
Example.

6.11.2014

Cauchy's integral formula for a disc.
Taylor's theorem.
Cauchy's formula for derivatives.
Examples.

7.11.2014

Morera's theorem.
Entire functions.
Liouville's theorem.
Fundamental Theorem of Algebra.
Zeros of analytic functions.
Maximum modulus theorem.

13.11.2014

A lemma of Schwarz.
On the winding number.

14.11.2014

Cauchy's Theorem (homology version) and its corollaries.
Examples.

20.11.2014

The proof of Cauchy's Theorem (homology version).
Deformation Theorem.
Example.

21.11.2014

Example.
On the extended complex plane.
Examples of Möbius transformations: translation, stretching, rotation, inversion.
The definition of a Möbius transformation.

27.11.2014

On Möbius transformations.

28.11.2014

More on Möbius transformations.

4.12.2014

On conformal mappings.

5.12.2014

More on conformal mappings.

Thank you for participating in the course.