Evolution and the Theory of Games
Fall 2014
Exercises 20+21

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Γ	S	H
S	$R+\delta\Gamma$	$\Delta\Gamma$
H	$r+\Delta\Gamma$	$r+\Delta\Gamma$

(a) Analyze the Iterated Stag Hunt with $0<r<R$ and $0<\delta<\Delta<1$.

S×S:
$E=R+\delta E$
$(1-\delta)E=R$
$E=\frac{R}{1-\delta}$

S×H:
$E=\Delta E$
$(1-\Delta)E=0$
$E=0$

H×S (or H×H):
$E=r+\Delta E$
$(1-\Delta)E=r$
$E=\frac{r}{1-\Delta}$

So, the overall payoff matrix (for the symmetric game with payoffs to the row player) is:

Γ	S	H
S	$\frac{R}{1-\delta}$	0
H	$\frac{r}{1-\Delta}$	$\frac{r}{1-\Delta}$

- S is an ESS if $\frac{R}{1-\delta}>\frac{r}{1-\Delta}$.
- H is an ESS if $\frac{r}{1-\Delta}>0$, i.e. always.

(b) Analyze $\Gamma=(\Gamma_1,\Gamma_2)$ with $0<r<R$ and $0<\delta<\Delta<1$.

Γ_1	S	H
S	$R+\delta\Gamma_2$	$\Delta\Gamma_1$
H	$r+\Delta\Gamma_1$	$r+\Delta\Gamma_1$

Γ_2	rest
rest	$\Delta\Gamma_1$

- The payoffs for the row player in Γ_1 are the same as in part (a) for S×H, H×S and H×H
- For S×S, $\begin{cases} E_1=R+\delta E_2 \\ E_2=\Delta E_1 \end{cases} \Rightarrow$ $E_1=R+\delta\Delta E_1$
  $(1-\delta\Delta)E_1=R$
  $E_1=\frac{R}{1-\delta\Delta}$

So, the overall payoff matrix is

Γ	(S, rest)	(H, rest)
(S, rest)	$\frac{R}{1-\delta\Delta}$	0
(H, rest)	$\frac{r}{1-\Delta}$	$\frac{r}{1-\Delta}$

- (S, rest) is an ESS if $\frac{R}{1-\delta\Delta}>\frac{r}{1-\Delta}$
- (H, rest) is an ESS if $\frac{r}{1-\Delta}>0$, i.e. always.

(c) Analyze $\Gamma = (\Gamma_1, \Gamma_2, \Gamma_3)$ with $0 < r < R$ and $0 < \delta < \Delta < 1$.

Γ_1	S	SR	H
S	$R + \delta\Gamma_1$	$R + \delta\Gamma_2$	$\Delta\Gamma_1$
SR	$R + \delta\Gamma_2$	$R + \delta\Gamma_3$	$\Delta\Gamma_1$
H	$r + \Delta\Gamma_1$	$r + \Delta\Gamma_1$	$r + \Delta\Gamma_1$

Γ_2	rest (SR player)
hunt a hare (S player)	$r + \Delta\Gamma_1$, $\Delta\Gamma_1$

Γ_3	rest (SR player)
rest (SR player)	$\Delta\Gamma_1$

In the overall payoff matrix, S×S, S×H, H×S and H×H yield the same payoffs as in part (a). SR×H yields the same payoff as S×H. H×SR yields the same payoff as H×S & H×H.

$\underline{S \times SR}$ $\quad$ $\underline{(\overset{\Gamma_1}{S}, \overset{\Gamma_2}{hare}) \times (SR, rest)}$

$E_1 = R + \delta E_2$ $\quad$ $E_2 = r + \Delta E_1$

$E_1 = R + \delta(r + \Delta E_1)$

$E_1 = R + \delta r + \delta\Delta E_1$

$(1-\delta\Delta)E_1 = R + \delta r$

$\boxed{E_1 = \frac{R+\delta r}{1-\delta\Delta}}$

$\underline{SR \times S}$ $\quad$ $\underline{(\overset{\Gamma_1}{SR}, \overset{\Gamma_2}{rest}) \times (S, hare)}$

$E_1 = R + \delta E_2$ $\quad$ $E_2 = \Delta E_1$

$E_1 = R + \delta\Delta E_1$

$(1-\delta\Delta)E_1 = R$

$\boxed{E_1 = \frac{R}{1-\delta\Delta}}$

$\underline{SR \times SR}$ $\quad$ $\underline{(\overset{\Gamma_1}{SR}, \overset{\Gamma_3}{rest}) \times (SR, rest)}$

$E_1 = R + \delta E_3$ $\quad$ $E_3 = \Delta E_1$

$E_1 = R + \delta\Delta E_1$

$(1-\delta\Delta)E_1 = R$

$\boxed{E_1 = \frac{R}{1-\delta\Delta}}$

So, the overall payoff matrix is

Γ	$(S, \binom{hare}{rest}, rest)$	$(SR, \binom{hare}{rest}, rest)$	$(H, \binom{hare}{rest}, rest)$
$(S, \binom{hare}{rest}, rest)$	$\frac{R}{1-\delta}$	$\frac{R+\delta r}{1-\delta\Delta}$	0
$(SR, \binom{hare}{rest}, rest)$	$\frac{R}{1-\delta\Delta}$	$\frac{R}{1-\delta\Delta}$	0
$(H, \binom{hare}{rest}, rest)$	$\frac{r}{1-\Delta}$	$\frac{r}{1-\Delta}$	$\frac{r}{1-\Delta}$

where the conditional strategy $\binom{a}{b}$ means "Play a if the row player in Γ_2, and play b if the column player in Γ_2."

- $(S, \binom{hare}{rest}, rest)$ is an ESS if $\frac{R}{1-\delta} > \frac{R}{1-\delta\Delta}$ and $\frac{R}{1-\delta} > \frac{r}{1-\Delta}$. The former is always satisfied, but the latter depends on the exact values of R, r, δ & Δ. So, this strategy is an ESS whenever $\frac{R}{1-\delta} > \frac{r}{1-\Delta}$.
- $(SR, \binom{hare}{rest}, rest)$ is an ESS if $\frac{R}{1-\delta\Delta} > \frac{R+\delta R}{1-\delta\Delta}$ and $\frac{r}{1-\Delta} < \frac{R}{1-\delta\Delta}$. The former is never satisfied, so this strategy is never an ESS.
- $(H, \binom{hare}{rest}, rest)$ is an ESS if $\frac{r}{1-\Delta} > 0$, which is always satisfied. So, this strategy is always an ESS.

21 Formulate IPD with TFT and sPav as a multi-stage game.

TFT	C D D	C D D	...
sPav	D D C	D D C	...

TFT	C C ...
TFT	C C ...

sPav	D C C C ...
sPav	D C C C ...

$S < P < R < T$

Γ_1	C (TFT)	D (sPav)
C (TFT)	$R + \delta\Gamma_1$	$S + \delta\Gamma_2^{(col)}$
D (sPav)	$T + \delta\Gamma_2^{(row)}$	$P + \delta\Gamma_4$

Γ_2	D (TFT)
D (sPav)	$P + \delta\Gamma_3^{(row)}, P + \delta\Gamma_3^{(col)}$

Γ_3	D (TFT)
C (sPav)	$S + \delta\Gamma_1, T + \delta\Gamma_1$

Γ_4	C (sPav)
C (sPav)	$R + \delta\Gamma_4$

(symmetric)

Strategy set (same for row + col players)

$\left\{ \left(C, \binom{D}{D}, \binom{C}{D}, \binom{C}{C}\right), \left(D, \binom{D}{D}, \binom{C}{D}, \binom{C}{C}\right) \right\}$

Note: $\binom{a}{b}$ is a conditional strategy wherein the player plays a if in the position of the row player and plays b if in the position of the column player.