EVOLUTION AND THE THEORY OF GAMES

STEFAN GERITZ, HELSINKI, 2011

Exercises 02-12-2011

18. Consider the Hawk-Dove game with payoff matrix

	H	D
H	$\frac{1}{2}(R-C)$, $\frac{1}{2}(R-C)$	R , 0
D	0 , R	$\frac{1}{2}R$, $\frac{1}{2}R$

(a) Analyze the *iterated* Hawk-Dove game for the strategies 'always hawk' (allH) and 'always dove' (allD), i.e., calculate the payoff matrix of the iterated game and determine whether and when any of the two strategies is an ESS.

(b) Idem for allH, allD and Bully (B), which starts with H in the first round, but then does the opposite of what its opponent did in the previous round.

(c) Idem for allH, allD and Retaliator (R), which starts with D in the first round, but then does the same as what its opponent did in the previous round.

(d) Idem for the strategies Bully (B) and Retaliator (R).

19. Consider two bulls fighting over the right to mate with the females of the herd. The game goes like this: First there is a lot of display of power and intention. Next, the two males run to one another on a collision course. Each can either swerve (S) or not swerve (NS). If neither swerves, both males end up with a terrible headache (or worse), and neither one wins. If both swerve, then there is no winner either, but at least no-one got hurt, and the game may be repeated again some other time. If only one of the males swerves, the other is the winner and gains the right to mate with all the females.

The prize of winning is R; the cost of display and charging is D; the cost of the headache (or worse) is C. The payoff matrix of the game thus is:

	S	NS
S	$-D$, $-D$	$-D$, $R-D$
NS	$R-D$, $-D$	$-C-D$, $-C-D$

Continued on the next page ...

(a) Analyze this game as a single shot (i.e., not repeated) game. Calculate the mixed ESS? What is the role of the "participation fee" D?

(b) Analyze the game as an iterated game in which the game is repeated with a probability $\delta \in (0, 1)$ and *only if both males swerved*. Consider only the pure strategies swerve (S), not swerve (NS) and the mixed strategy (M) found in **(a)**. What is the role of the "participation fee" D now?

(c)* Do you think that there are better mixed strategies than M? Analyze the game as in **(b)**, with the pure strategies swerve (S), not swerve (NS) and a general mixed strategy (G). When is G better than M and in what sense?

(d) Suppose we include for another strategy altogether: the 'opportunist' (O) who does not fight at all, but attempts to mate with females while other males are busy with their fights. Assume the opportunistic strategy has an expected payoff B over the entire iterated game. When is the opportunist an ESS? What would that mean, i.e., is there any sense in this?

(N.B., this game is commonly known as the Chicken Game in which two drivers drive towards each other on a collision course: one must swerve, or both may die in the crash, but if one driver swerves and the other does not, the one who swerved will be called a "chicken", meaning a coward.)