Department of mathematics and statistics
Differential Equations II
Compensating Course Exam 13.1.2015

Remark. Use of an abstract page of the size A4 is allowed to a candidate.

1. Give in a real form a fundamental solution set in $\mathbf{R}$ to the following homogeneous pair:

$$\dot{\mathbf{x}}(t) = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix} \mathbf{x}(t).$$

2. Give a general solution to the following system:

$$\dot{\mathbf{x}}(t) = \begin{bmatrix} 0 & 1/2 \\ 2 & 0 \end{bmatrix} \mathbf{x}(t) + \begin{bmatrix} 3/2 \\ 4 \end{bmatrix}.$$

3. (a, 3 points) Reduce the differential equation

$$\ddot{x} = -2\dot{x} + x^2 + x - 2$$

of second order to an equivalent pair of first order.

(b, 3 points) Solve the critical points of that pair and determine their qualities (stable or unstable) by using the Poincaré theorem.

4. Consider the initial value problem

$$y' = e^{xy}(y^2 - 2), \quad y(0) = -1,$$

of one differential equation. Let $y : I \to \mathbf{R}$ a (maximal) solution to it.

(a, 3 points) Show that y is a bounded function.

(b, 3 points) Show that $I = \mathbf{R}$, i.e. that the solution y exists for all $x \in \mathbf{R}$.