Department of mathematics and statistics
Differential Equations II
Course Exam 16.12.2014

Remark. Use of an abstract page of the size A4 is allowed to a candidate.

1. Give a fundamental solution set in $\mathbf{R}$ to the following homogeneous system:

$$\dot{\mathbf{x}}(t) = \begin{bmatrix} -2 & -1 \\ 1 & 0 \end{bmatrix} \mathbf{x}(t).$$

2. Give a general solution to the following system:

$$\dot{\mathbf{x}}(t) = \begin{bmatrix} 1 & 1 \\ 4 & 1 \end{bmatrix} \mathbf{x}(t) + \begin{bmatrix} 2e^t \\ -3e^t \end{bmatrix}.$$

3. (a, 3 points) Reduce the differential equation

$$\ddot{x} - 2x\cos(\dot{x}) + \dot{x} - x^2 + 3 = 0$$

of second order to an equivalent pair of first order.

(b, 3 points) Solve the critical points of this pair and determine their qualities by using the Poincaré theorem (stable or unstable).

4. Consider the initial value problem

$$y' = \frac{2\sin^2(xy)}{x+1}, \quad y(0) = 1.$$

of one differential equation. Let $y : I \to \mathbf{R}$ a (maximal) solution to it. Show that $I =]-1, \infty[$, i.e. that the solution y exists for all $x \in]-1, \infty[$.