**Differential Equations II**
Exercise 6, fall 2014

**Remark.** Problem 3 has been again changed 3.12.

1. Determine critical points of the following autonomous system and determine also their qualities (stable or unstable):

$$\dot{x} = 2y - 2$$
$$\dot{y} = -x - 2y.$$

2. Determine critical points of the following autonomous system and determine also their qualities (stable or unstable):

$$\dot{x} = -2x + y + 6$$
$$\dot{y} = -x + 2y - 3.$$

3. Determine critical points of the following autonomous system and determine also their qualities (stable or unstable):

$$\dot{x} = 3x^2 - y$$
$$\dot{y} = 3 + 6x^2 - y^2.$$

4. Determine critical points of the following autonomous system:

$$\dot{x} = (x+1)(y-2)$$
$$\dot{y} = x^2 - x - 2.$$

What does the Poincaré Theorem (Theorem 6.5 in the material) tell about qualities of the critical points? Determine also trajectories of the system.

5. Consider one differential equation of 1st order (so the dimension of problem is one)

$$y'(x)^2 + y(x)^2 = 1. \qquad (*)$$

(a) Give two solutions to $(*)$ such that both satisfy $y(\pi/4) = 0$.

(b) Let $B = \{(x, y) \in \mathbf{R}^2 \mid (x - \pi/4)^2 + y^2 < 1/4\}$. Show that your solutions are the only ones in $B$ to the IVP $(*)$, $y(\pi/4) = 0$.

A tip (b). The EU Theorem.

6. Consider the initial value problem of one differential equation

$$y'(x) = \frac{\sin y + 1}{x^2 - 1}, \quad y(0) = 0.$$

Let $y : I \to \mathbf{R}$ be a (maximal) solution to it. Show that $I = ]-1, 1[$, i.e. the solution $y$ exists in the interval $]-1, 1[$ (the equation is not even defined when $x = \pm 1$).

A tip. Either the global version of the EU Theorem or Theorem 4.7 in the material.