Differential Equations II

Exercise 5, fall 2014

1. Find a fundamental solution set in $\mathbf{R}$ to the following homogeneous system by the matrix method, which uses generalized eigenvectors:

$$\dot{\mathbf{x}}(t) = A\mathbf{x}(t), \quad A = \begin{bmatrix} 5 & -4 & 0 \\ 1 & 0 & 2 \\ 0 & 2 & 5 \end{bmatrix} \in \mathbf{R}^{3\times 3}.$$

2. Solve the linear system

$$\dot{\mathbf{x}}(t) = A\mathbf{x}(t) + \mathbf{f}(t), \quad A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}, \quad \mathbf{f}(t) = \begin{bmatrix} -2 \\ 3 \end{bmatrix},$$

by using an appropriate direct try.

3. Solve the linear system

$$\dot{\mathbf{x}}(t) = A\mathbf{x}(t) + \mathbf{f}(t), \quad A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}, \quad \mathbf{f}(t) = \begin{bmatrix} -\sin t \\ \cos t \end{bmatrix},$$

by using variation. Note that A is the same one as in the previous problem.

4. Solve the linear system

$$\dot{\mathbf{x}}(t) = A\mathbf{x}(t) + \mathbf{f}(t), \quad A = \begin{bmatrix} 1 & 1 \\ 4 & 1 \end{bmatrix}, \quad \mathbf{f}(t) = \begin{bmatrix} -\cos t \\ -\sin t \end{bmatrix},$$

by using an appropriate direct try.

A tip. A straightforward calculation yields a linear algebraic equation system of size 4×4.

5. Solve the linear system

$$\dot{\mathbf{x}}(t) = A\mathbf{x}(t) + \mathbf{f}(t), \quad A = \begin{bmatrix} 1 & 1 \\ 4 & 1 \end{bmatrix}, \quad \mathbf{f}(t) = e^{3t}\begin{bmatrix} 3 \\ -2 \end{bmatrix},$$

by using variation. Note that A is the same one as in the previous problem. Additionally, what direct try would work here?

6. Solve the linear system

$$\dot{\mathbf{x}}(t) = A\mathbf{x}(t) + \mathbf{f}(t), \quad A = \begin{bmatrix} 5 & -4 & 0 \\ 1 & 0 & 2 \\ 0 & 2 & 5 \end{bmatrix}, \quad \mathbf{f}(t) = \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix},$$

by using an appropriate direct try. A is the same one as in the problem 1.