Differential Equations II

Exercise 4, fall 2014

1. Form a fundamental solution set in $\mathbf{R}$ to the homogeneous system $\dot{\mathbf{x}}(t) = A\mathbf{x}(t)$, when

$$A = \begin{bmatrix} 2 & 4 \\ 1 & -1 \end{bmatrix} \in \mathbf{R}^{2\times 2}.$$

2. Form a fundamental solution set in $\mathbf{R}$ to the homogeneous system $\dot{\mathbf{x}}(t) = A\mathbf{x}(t)$, when

$$A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 2 & 1 \end{bmatrix} \in \mathbf{R}^{3\times 3}.$$

3. Form a fundamental matrix in $\mathbf{R}$ to the homogeneous system $\dot{\mathbf{x}}(t) = A\mathbf{x}(t)$, when

$$A = \begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix} \in \mathbf{R}^{2\times 2}.$$

Give a real matrix.

4. Form a fundamental solution set in $\mathbf{R}$ to the homogeneous system $\dot{\mathbf{x}}(t) = A\mathbf{x}(t)$, when

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \in \mathbf{R}^{3\times 3}.$$

Give a set as real functions.

5. Find a fundamental solution set in $\mathbf{R}$ to the following homogeneous system by the matrix method, which uses generalized eigenvectors:

$$\dot{\mathbf{x}}(t) = \begin{bmatrix} -1 & 1 \\ -1 & -3 \end{bmatrix} \mathbf{x}(t).$$

A tip. Equations (5.31) and (5.32) in the lecture material.

6. Find a fundamental solution set in $\mathbf{R}$ to the following homogeneous system by the matrix method, which uses generalized eigenvectors:

$$\dot{\mathbf{x}}(t) = A\mathbf{x}(t), \quad A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 0 & -1 & 1 \end{bmatrix} \in \mathbf{R}^{3\times 3}.$$