Differential Equations II

Exercise 3, fall 2014

1. Let the function $\mathbf{z}(t) = (x(t), y(t))$ be a (maximal) solution to the IVP

$$\dot{\mathbf{z}}(t) = \begin{bmatrix} \dot{x}(t) \\ \dot{y}(t) \end{bmatrix} = \begin{bmatrix} (t+y)(x^2+y^2-1) \\ (t-x)(x^2+y^2-1) \end{bmatrix}, \quad \mathbf{z}(0) = (1/2, 1/2).$$

Show by using an analogy to Theorem 4.7 (”Graph Goes Away Theorem of Systems”), that $\mathbf{z}$ is defined in the whole $\mathbf{R}$.

Tips. Trivial solutions. Can $\|\mathbf{z}(t)\| \geq 1$ hold to the solution? A reason. A figure in the txy cordinate system can be helpful.

2. The general solution in Problem 1 of Exercise 1 has three parameters. Show exactly, that that solution gives all the solutions to the DE in question.

Tips. Theorem 5.4, or if you want to reduce to a system, Theorem 5.3 or 5.5 or perhaps most simply 5.9.

3. Write the following 2×2 homogeneous system traditionally ”open” and solve it:

$$\dot{\mathbf{z}}(t) = \begin{bmatrix} 2t & 3t^2 \\ 0 & 2t \end{bmatrix} \mathbf{z}(t).$$

How many free parameters do you have? Give some fundamental solution set to the system.

4. Show that the function pair $\big(\mathbf{x}_1(t), \mathbf{x}_2(t)\big) = \Big([1 \ e^t]^T,\ [e^{-t} \ 2]^T\Big)$ is a fundamental solution set in $\mathbf{R}$ to the linear 2×2 homogeneous system

$$\dot{\mathbf{x}}(t) = \begin{bmatrix} 1 & -e^{-t} \\ 2e^t & -1 \end{bmatrix} \mathbf{x}(t).$$

5. Write the following linear 2×2 system traditionally ”open” and solve it then by the elimination method:

$$\dot{\mathbf{z}}(t) = \begin{bmatrix} 1 & 1 \\ 3 & -1 \end{bmatrix} \mathbf{z}(t) + 4 \begin{bmatrix} 1 \\ t \end{bmatrix}.$$

6. Find by the matrix method a fundamental solution set to the following 3×3 homogeneous system:

$$\dot{\mathbf{x}}(t) = \begin{bmatrix} 1 & -1 & 4 \\ 3 & 2 & -1 \\ 2 & 1 & -1 \end{bmatrix} \mathbf{x}(t).$$