Differential Equations II
Exercise 2, fall 2014

1. Show that the function

$$f(x,y) = x\sqrt[3]{y+1}$$

is not uniformly Lipschitz continuous with respect to y in any rectangle K, where $(x,-1) \in K$ for some $x \in \mathbf{R} \setminus \{0\}$ (see Problem 4 in Exercise 1).

2. When considering a planetary orbit in the two body problem, for the angle function $\theta = \theta(t)$ we got the following separable DE

$$\dot{\theta}(t) = c_1\Big(Kc_1^{-2} + c\cos\big(\theta(t)-\delta\big)\Big)^2,$$

where t is time and δ, c, c_1 and K are constants. Show by the global EU theorem 4.6 (the number in the lecture material) that a (maximal) solution θ to the DE is defined in the whole $\mathbf{R}$.

Tips. You can choose $y(0) = 0$ as an initial condition (any other as well), the Mean Value Theorem.

3. Show by the global EU theorem 4.6 that a (maximal) solution y to the IVP

$$y' = e^x \sin x \cos y, \quad y(0) = 0,$$

is defined in the whole $\mathbf{R}$.

A tip. The Mean Value Theorem.

4. The same problem as 2, but use now the theorem 4.7 ("Graph Goes Away Theorem").

Tips. By the equation $\theta(t) - \theta(0) = \int_0^t \dot{\theta}(\tau)\, d\tau$ you can obtain limits to $\theta(t)$. Then use an increasing sequence of rectangles as compact sets.

5. (a) Reduce the DE $y^{(3)} + x^3\cos(y'') + \cos(x)y = \sin x$ to a system of first order.

(b) Reduce the following system of second order to a system of first order and of a normal form:

$$m\ddot{x}_1(t) + 2kx_1(t) - kx_2(t) = 0$$
$$m\ddot{x}_2(t) - kx_1(t) + 2kx_2(t) = 0.$$

6. (a) Reduce the following system to a system of first order and of a normal form:

$$\ddot{x} = f(t, x, y, \dot{x})$$
$$\dot{y} = g(t, x, y).$$

(b) What about if the second equation was

$$\ddot{x} = f(t, x, y, \dot{x}, \dot{y})?$$