

Remark. Use of an abstract page of the size A4 is allowed to a candidate.

1. Solve the initial value problem

$$y' = \cos(x)e^{-y}, \quad y(0) = 0.$$

2. Solve implicitly the differential equation

$$2x \sin y + (3y^2 + x^2 \cos y)y' = 0.$$

3. Solve the differential equation

$$y'' - 2y' + 2y = 10 \sin(2x).$$

4. (a, 3 points) Solve the differential equation

$$xy' - y = 5y^3. \quad (*)$$

(b, 3 points) Determine all the points $(x_0, y_0) \in \mathbf{R}^2$ such that any solution to equation (*) does not contact them. Give reasons.