Differential Equation I
Exercise 6, fall 2014

1. Solve the equation

$$3y'' + 2y' + y = 0.$$

2. Solve the equation

$$\ddot{x} + 9x = 4t - \sin t$$

by using an appropriate direct try (the method of undetermined coefficients).

3. Solve the equation

$$\ddot{x} - 9x = e^{-3t}$$

by variation. So afterwards, what would have been an appropriate form for a direct try?

4. Solve on the interval $]0, \infty[$ the homogeneous equation

$$y'' + \frac{1}{x}\,y' + \frac{1}{x^2}\,y = 0,$$

(a) by a direct try $y(x) = x^a$, where a is a parameter,
(b) by changing appropriately the free variable x, that is $x = x(t)$.
Tips. (a) The Euler's formula. (b) A new unknown function is then $z(t) = y(x(t))$. Replace y, y' and y''.

5. Briefly can be seen that e^{2x} is a solution in $\mathbf{R}$ to the homogeneous equation

$$(x-2)\,y'' - (4x-7)\,y' + (4x-6)\,y = 0. \qquad *$$

Find another independent solution by using the method of variation and give also to $*$ a general solution in $\mathbf{R}$.

6. Extend the problem 5:
(a) Does your general solution give all the solutions to equation $*$ in $\mathbf{R}$? Give reasons.
(b) Does the IVP, $*$ with $y(2) = 1,\ y'(2) = 0$, have a solution? A reason connected to theory?
Tips. (a) You shall need the general theory of homogeneous equations. Continuity. A more difficult question: why does continuity concern also a second derivative of a solution function?