Differential Equation I

Excercise 5, fall 2014

1. Solve (implicitly) the equation

$$\frac{dy}{dx} = \frac{-3x + y + 4}{x + 3y + 2}.$$

2. Study by the Wronskian determinant, which of the following function pairs (y_1, y_2) can noway be a fundamental solution set on the interval $]0, \infty[$ for any homogeneous differential equation of second order:

$$(x^3, x), \quad (\sin 2x, -\cos 2x), \quad (\sin 2x, \cos x).$$

3. Consider the following function pairs (y_1, y_2):

$$\big(7e^{-2x}, e^{-2x}\big), \quad \big(-xe^{-2x}, -e^{-2x}\big), \quad \big(e^{-x}, -e^{-2x}\big).$$

(a) Determine their Wronskian determinants at the points $x \in \mathbf{R}$.

(b) Which of the pairs do form a fundamental solution set on the interval $\mathbf{R}$ for the equation $y'' + 4y' + 4y = 0$? Give reasons.

4. Find by the direct try $y(x) = x^a$, where a is a parameter, a fundamental solution set on the interval $I =]0, \infty[$ for the homogeneous equation

$$y'' + \frac{4}{x}\, y' - \frac{4}{x^2}\, y = 0.$$

5. Solve the equations

$$(a) \quad 3\ddot{x} + \dot{x} - 2x = 0, \quad (b) \quad \ddot{x} + 4x = 4\dot{x}.$$

6. Solve the IVP

$$9y'' + 12y' + 4y = 0, \quad y(0) = 3,\ y'(0) = -1.$$