Differential Equation I
Excercise 4, fall 2014

1. A bacteria population is modeled by the exponential law of population growth. Suppose mass of bacteria initially is $2\,mg$ and after a week it is $100\,g$.
(a) Determine the Malthusian parameter of the model (also a unit).
(b) When is the population 10000 times that in the beginning?

2. Solve the IVP

$$\dot{x} + 4x = 4t\sqrt{x}, \quad x(0) = 1/4.$$

Briefly, is a solution defined on the whole $\mathbf{R}$?
A tip. You may need partial integration.

3. In a village of 1000 inhabitants it is realized duck influence with a duration parameter $\alpha = 0.60\,vrk^{-1}$ and an infection intensity $\beta = 10^{-3}\,vrk^{-1}$. In a consideration interval nobody dies and nobody is born - thus, the infection is modeled by the SIS -model.
(a) Write a DE-pair of the model and from it by elimination obtained logistic equation.
(b) Solve this equation as an equation of Bernoull's type and show by a solution that a number of infected ones just increases (that is, an epidemic arises), when in the beginning there is only 10 infected.
(c) How can you directly see that increase?

4. Consider the SIR-model of infectious diseases, especially the pair (2.16),

$$\frac{ds}{dt}(t) = -\alpha R_0 s(t)i(t), \quad \frac{di}{dt}(t) = \alpha R_0 s(t)i(t) - \alpha i(t).$$

Suppose $0 < s(0), i(0) < 1$. Show that $i(t), s(t) > 0$ for all $t \geq 0$.

Tips. You can suppose the solution exists on $[0, \infty[$. For any solution functions $i(t)$ and $s(t)$ you can alternately fix them and apply the existence and uniqueness Theorem 1.2 separately to the equations (2.16).

5. Extend the problem 4: show that

$$(a) \quad \exists s_\infty = \lim_{t\to\infty} s(t) > 0, \quad (b) \quad \exists i_\infty = \lim_{t\to\infty} i(t) = 0.$$

Tips. Use Problem 4 and Equation (2.18), finally Lemma 2.1.

6. Solve, at least implicitly, the differential equations

$$(a) \quad y' = (x - y + 1)^2, \quad (b) \quad x^3 - xy^2y' + y^3 = 0.$$