Differential Equation I
Excercise 3, fall 2014

1. Solve implicitly the DE

$$2xy + (2y^2 + x^2)\, y' = 0.$$

2. Find an integrating factor for the DE

$$(x - y^2/2) + xyy' = 0$$

and solve it. Can a solution be explicitly represented?

3. A water tank is of a size 16m $\times$ 10m $\times$ 3m. Initially it is full of 3 per cent brine.

(a) Brine of 1 per cent enters the tank at the rate $1\, m^3/min$, and the well stirred brine leaves the tank at the same rate. When does the concentration of salt in the tank descend to the level of 2 per cent?

(b) Otherwise the same situation but brine leaves the tank at the rate $1.5\, m^3/min$.

4. Find the curves of the form $y = y(x)$ such that for all x_0 a tangent line, set at $\big(x_0, y(x_0)\big)$, intersect the x-axis at $\big(x_0 + x_0^2/k,\, 0\big)$, where $k \neq 0$ is a constant.

5. A fish population of a lake was estimated to be 10000 individuals in 1990 and 5000 individuals in 2000. Let us model the fish population $p(t)$ by the logistic equation $\dot{p}(t) = rp(t)\big(1 - p(t)/K\big)$. Suppose that the parameter r has an estimated value 0.1 (when the unit of time is a year), but the tolerance K of environment is unknown.

(a) Determine K, (b) predict the population in 2010.

Remark. If also r is unknown, at least three known values of population are needed. Consideration yields then a system of equations (an usual system, not differential equations) that (mostly) has to be solve by some numerical method.

6. Let temperatures of a body and its environment be $T_1 = T_1(t)$ and $T_2 = T_2(t)$ as functions of time t, and suppose they interact so that at each moment their rates of change are proportional to the difference $T_1(t) - T_2(t)$ between temperatures, $a < 0$ and $b > 0$ as proportional constants (so called the Newton's law of cooling).

(a) Form a pair of differential equations for the functions T_1 and T_2.

(b) Can you solve that?

A tip. (b) Eliminate the pair to one equation.