Differential Equations I
Exercise 2, fall 2014

1. Solve the differential equations (DE)

$$(a) \quad y' = 2x + y/x, \quad (b) \quad y' + (\cos x)y = \cos x.$$

2. Solve the DE

$$2y + 3 + (2x - 1)y' = 0$$

(a) as a linear, (b) as a separable equation.

3. If possible, reduce the following ones to ordinary differential equations, and if you succeed in it, also solve:

$$(a) \quad xy(x) + \int_0^{x^2} ty(t)\,dt = 1, \quad (b) \quad xy(x) + \int_0^{x} ty(t)\,dt = 1.$$

4. Solve the DE

$$(x - 2)y' - y = 2(x - 2)^3$$

by the initial conditions

$$(a) \quad y(0) = 0, \quad (b) \quad y(2) = 0, \quad (c) \quad y(2) = 1.$$

How can solutions be understood from a point of view of the existence and uniqueness theorem (Theorem 1.2)?

5. Consider the DE

$$-2xy^2 + (1 + 2y)y' = 0.$$

(a) What is wrong with the following deduction:

$$\begin{aligned}&-2xy^2 + (1 + 2y)y' = 0 \Leftrightarrow c = \int 0 = \int -2xy^2 + \int y' + \int 2yy' \\ &\Leftrightarrow -y^2 \int 2x - 2x \int y^2 + y + y^2 = -x^2y^2 - (2/3)xy^3 + y + y^2 = c, \quad c \in \mathbf{R},\end{aligned}$$

where the last equation would be an implicit solution.
(b) Solve correctly the equation (implicitly).

6. Show that the DE

$$y^{-1} + (2y - xy^{-2})\, y' = 0$$

is exact and solve it. Give explicitly an inverse function of solution.