Differential Equations I
Exercise 1, fall 2014

1. Which of the following ones are ordinary differential equations (DE)? Name also an unknown function and give the order of DE. If possible, reduce the DE also to a normal form.

$$(a)\quad \ddot{x} = (\dot{x}-2)^2 - tx, \qquad (b)\quad xy + \frac{d}{dx}(xy) = x^2,$$
$$(c)\quad xyz = \frac{\partial z}{\partial y} - 2xy + yz, \qquad (d)\quad y'(x) = y(x+1),$$
$$(e)\quad x^2\sin(y') + y^{(4)}/y = \cos x^3, \qquad (f)\quad x\sin(\dot{x})/t = t/\dot{x}.$$

Remark. In equation (d) all in brackets belongs to an argument of the function.

2. Show that the functions

$$(a)\quad y = Ae^x, \qquad (b)\quad y = e^{-x}(A\sin x + B\cos x), \qquad (c)\quad y = A\exp(x^2/2)$$

(A and B constants) are solutions in $\mathbf{R}$ to the differential eguations

$$(a)\quad y' - y = 0, \qquad (b)\quad y'' + 2y' + 2y = 0, \qquad (c)\quad y'' - xy' - y = 0.$$

3. Show that the function $y = \sin x$ is a solution to the initial value problem (IVP)

$$y' = \sqrt{1-y^2}, \quad y(0) = 0.$$

What is a maximal (connected) solution interval? Notice that the square root is nonnegative.

4. (a) Let y and z be two different solutions to DE $2y' + x - \sin y = 0$. Show that they don't intersect, that is, there is no x_0 where $y(x_0) = z(x_0)$.
A tip. Use a theoretical result.
(b) Can there exist many solutions to IVP $y' = \sqrt{1-y^2}, \quad y(\pi/2) = 1$? A reason from a theoretical point of view (briefly)?

5. Represent as partial fractions the function

$$f(x) = \frac{5}{(x+1)(2x-3)} = \frac{A}{x+1} + \frac{B}{2x-3},$$

that is, determine the constants A and B, and also integrate the function.
A tip. Multiply the representation on either side by factors of the denominator.

6. Solve separable ones of the following equations:

$$(a) \quad y' = \tan(xy - y), \quad (b) \quad y' = 2x + 2xy, \quad (c) \quad y' = (y+1)(2y-3).$$

Nonseparability needs no proof.