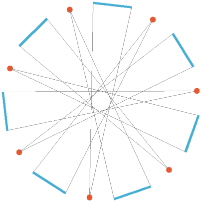
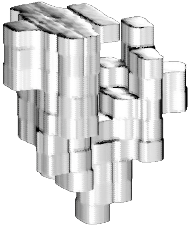
X-ray tomography





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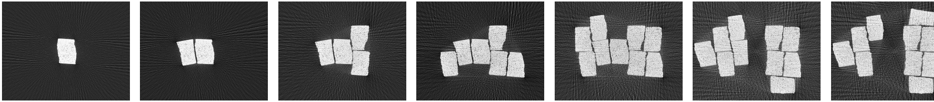
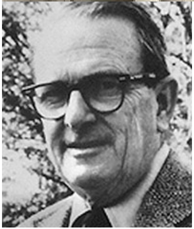
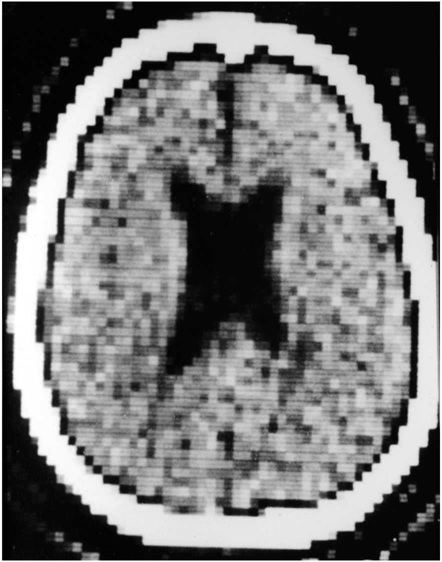


Godfrey Hounsfield and Allan McLeod Cormack were the first to develop X-ray tomography



Cormack (left) and Hounsfield (top) received Nobel prizes in 1979. Right: an early tomographic image.



Reconstruction of a function from its line integrals was first invented by Johann Radon in 1917

This is Radon's original formula:

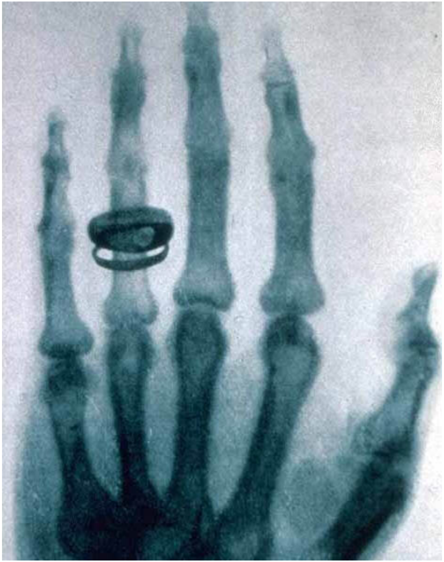
$$f(P) = -\frac{1}{\pi}\int_0^\infty \frac{d\overline{F_p}(q)}{q}.$$

Almost all tomography devices use *Filtered Back-Projection* (FBP), an algorithm derived from the above formula. See [Natterer 1986].

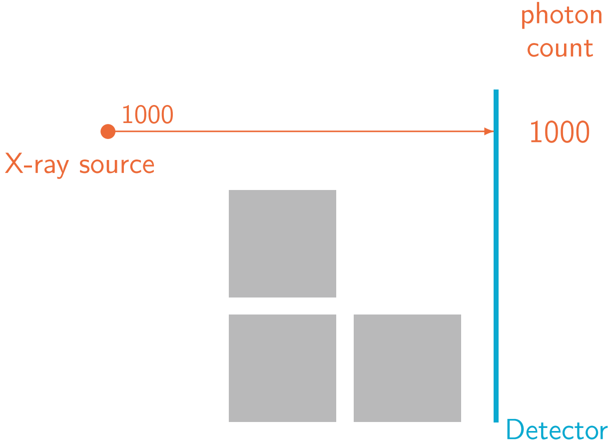
We consider sparsely sampled data. FBP is not optimal for such data, and we need to use for example Total Variation (TV) regularization:

$$\text{minimize}\{\|Af - m\|_2^2 + \alpha\|\nabla f\|_1\}.$$

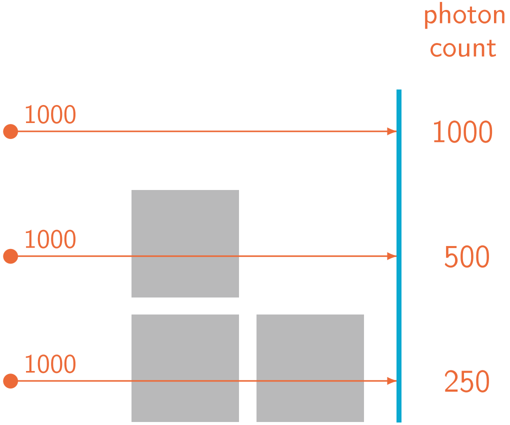
Wilhelm Conrad Röntgen invented X-rays and was awarded the first Nobel Prize in Physics in 1901



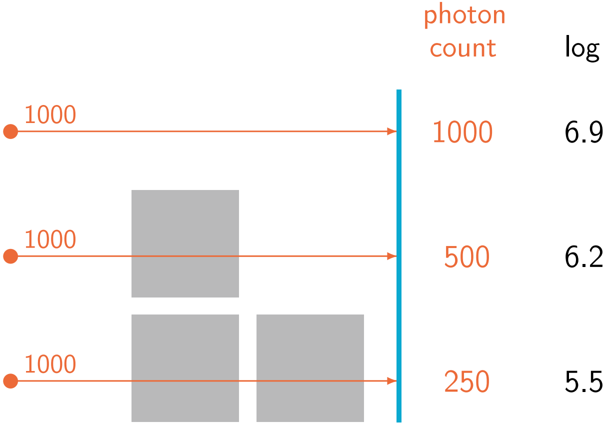
A digital X-ray detector counts how many photons arrive at each pixel



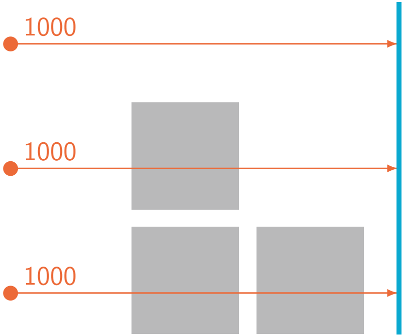
Adding material between the source and detector reveals the exponential X-ray attenuation law



We take logarithm of the photon counts to compensate for the exponential attenuation law

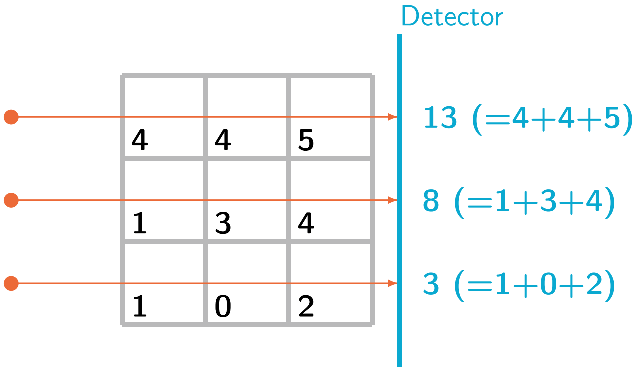


Final calibration step is to subtract the logarithms from the empty space value (here 6.9)

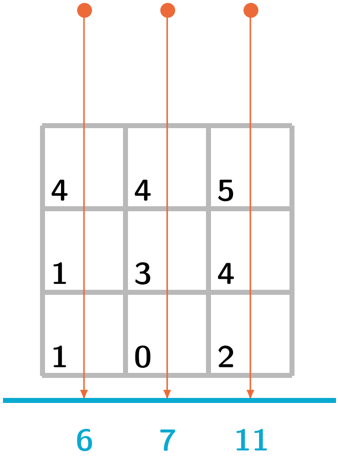


photon count	log	line integral
1000	6.9	**0.0**
500	6.2	**0.7**
250	5.5	**1.4**

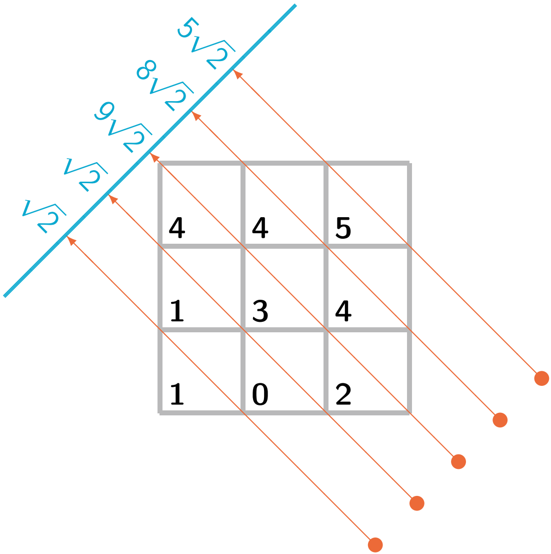
A projection image is produced by parallel X-rays and several detector pixels (here three pixels)



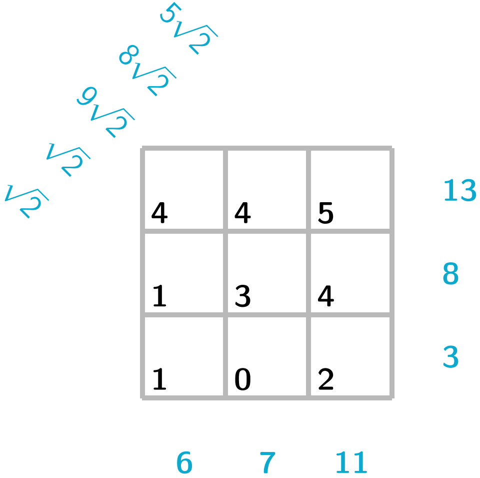
For tomographic imaging it is essential to record projection images from different directions



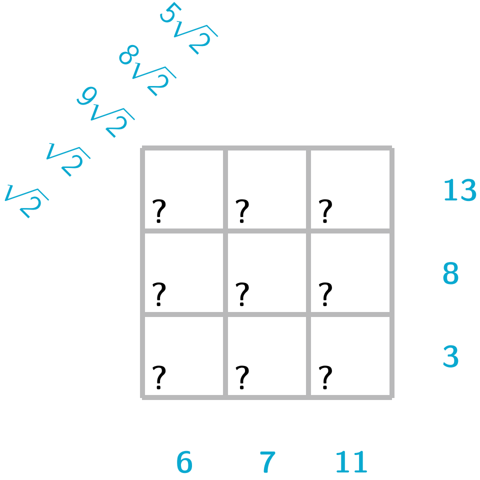
The length of X-rays traveling inside each pixel is important, thus here the square roots



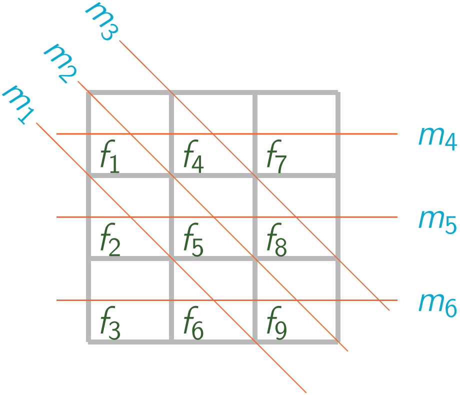
The direct problem of tomography is to find the projection images from known tissue



The inverse problem of tomography is to reconstruct the interior from X-ray data



We write the reconstruction problem in matrix form

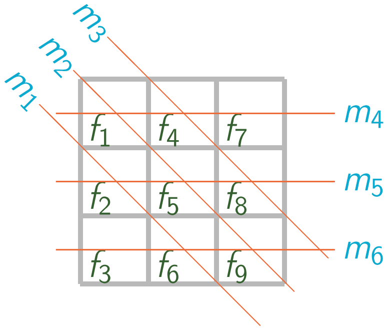


$$f = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \\ f_5 \\ f_6 \\ f_7 \\ f_8 \\ f_9 \end{bmatrix}, \quad m = \begin{bmatrix} m_1 \\ m_2 \\ m_3 \\ m_4 \\ m_5 \\ m_6 \end{bmatrix},$$

Measurement model: $m = Af + \varepsilon$

This is the matrix equation related to the above measurement

$$
\begin{bmatrix} m_1 \\ m_2 \\ m_3 \\ m_4 \\ m_5 \\ m_6 \end{bmatrix}
=
\begin{bmatrix}
0 & \sqrt{2} & 0 & 0 & 0 & \sqrt{2} & 0 & 0 & 0 \\
\sqrt{2} & 0 & 0 & 0 & \sqrt{2} & 0 & 0 & 0 & \sqrt{2} \\
0 & 0 & 0 & \sqrt{2} & 0 & 0 & 0 & \sqrt{2} & 0 \\
1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\
0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\
0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \\ f_5 \\ f_6 \\ f_7 \\ f_8 \\ f_9 \end{bmatrix}
+
\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \\ \varepsilon_5 \\ \varepsilon_6 \end{bmatrix}
$$



The VT device was developed in 2001–2012 by

Nuutti Hyvönen
Seppo Järvenpää
Jari Kaipio
Martti Kalke
Petri Koistinen
Ville Kolehmainen
Matti Lassas
Jan Moberg
Kati Niinimäki
Juha Pirttilä
Maaria Rantala
Eero Saksman
Henri Setälä
Erkki Somersalo
Antti Vanne
Simopekka Vänskä
Richard L. Webber

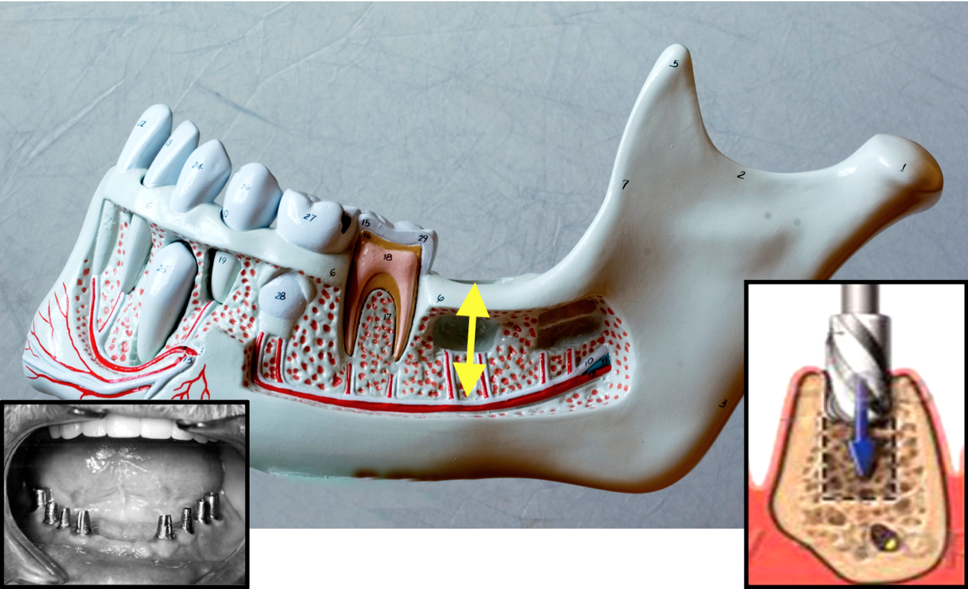
INSTRUMENTARIUM imaging

GE Healthcare

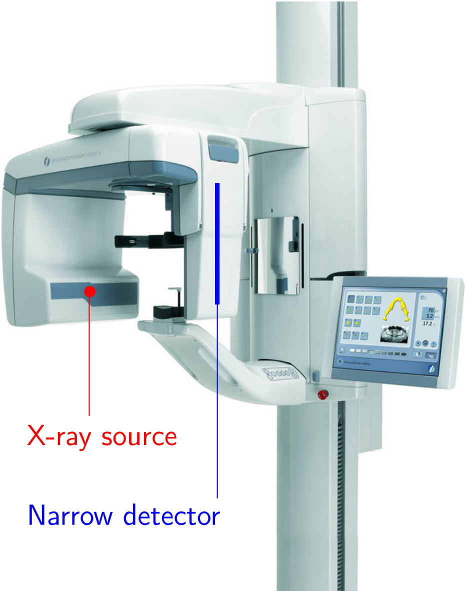
PALODEX GROUP

Tekes

Application: dental implant planning, where a missing tooth is replaced with an implant



Nowadays, a digital panoramic imaging device is standard equipment at dental clinics



A panoramic dental image offers a general overview showing all teeth and other dento-maxillofacial structures simultaneously.

Panoramic images are not suitable for dental implant planning because of unavoidable geometric distortion.

We reprogram the panoramic X-ray device so that it collects projection data by scanning

(Loading video)

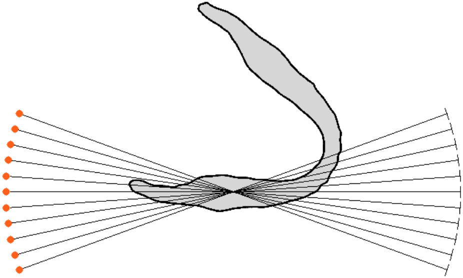
We reprogram the panoramic X-ray device so that it collects projection data by scanning

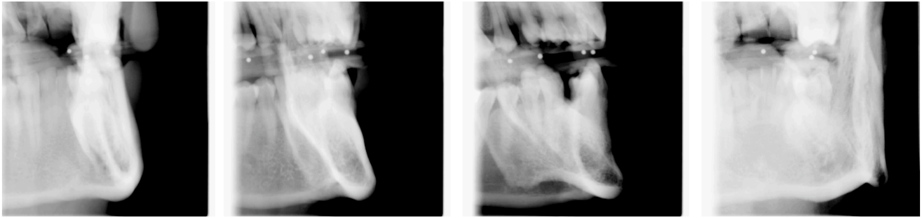
Number of projection images: **11**

Angle of view: **40 degrees**

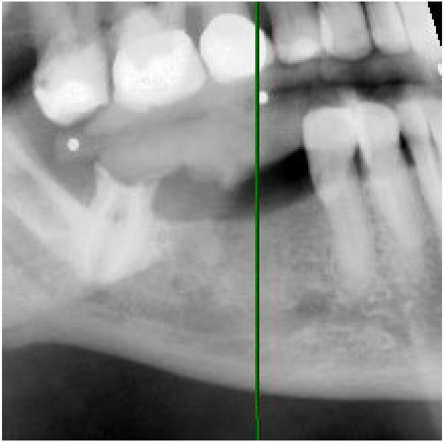
Image size: **1000×1000 pixels**

The unknown vector f has **7 000 000** elements.

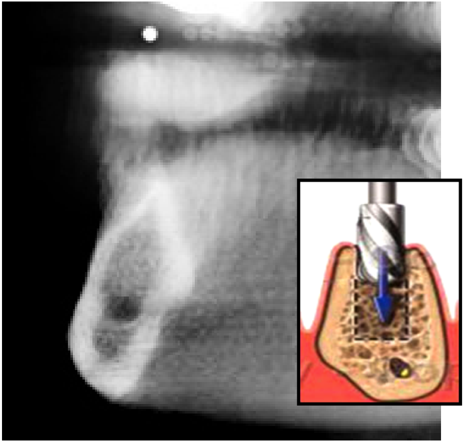




Here are example images of an actual patient: navigation image (left) and desired slice (right).



Kolehmainen, Vanne, S, Järvenpää, Kaipio, Lassas & Kalke **2006**, Kolehmainen, Lassas & S **2008**



Cederlund, Kalke & Welander **2009**, Hyvönen, Kalke, Lassas, Setälä & S **2010**, U.S. patent 7269241

The radiation dose of the VT device is lowest among 3D dental imaging modalities

Modality	μSv
Head CT	2100
CB Mercuray	558
i-Cat	193
NewTom 3G	59
VT device	**13**

Ludlow, Davies-Ludlow, Brooks & Howerton **2006**