VEKTORIANALYYSI / CALCULUS OF SEVERAL VARIABLES
LASKUHARJOITUS 1 / EXERCISE 1
SYKSY 2013 / AUTUMN 2013

1. Piirrä $\mathbb{R}^3$:n koordinaatistoon joukot
a) $A := B((0,3,3),3)$,

b) $B := \Big\{(x,y,z) \mid 0 \le x \le 2,\ 1 \le y \le 4,\ 1 \le z \le 4,\Big\}$,
c) joukko $B \cap B_1$, kun
$B_1 := \Big\{(x,y,z) \mid z < 5 - y\Big\}$

2. Olkoon $\bar{a} = (2,0,2) \in \mathbb{R}^3$, ja olkoon $\bar{y} \in \mathbb{R}^3$ vektori, jolle $|\bar{y}| = 1$. Mikä on vektorin $\bar{a} + \bar{y}$
a) suurin mahdollinen pituus,
b) pienin mahdollinen pituus?
Etsi jokin $\bar{y} \in \mathbb{R}^3$, $|\bar{y}| = 1$, joka on kohtisuorassa vektoria $\bar{a}$ vastaan.

3. Muodosta jonon $(\bar{x}^{(m)})_{m=1}^{\infty} \subset \mathbb{R}^3$, missä

$$\begin{aligned}
\text{a)}\quad \bar{x}^{(m)} &= (e^{-m^2+m}, 2 + m^{-2}, me^{-m}) \\
\text{b)}\quad \bar{x}^{(m)} &= (e^{-m^2+m}, 2 + m^{-2}, me^{-m}) + (1, e^{-m}, 2) \\
\text{c)}\quad \bar{x}^{(m)} &= (m+2)\big(e^{-m^2+m}, 2 + m^{-2}, me^{-m}\big)
\end{aligned}$$

komponentti- eli koordinaattijonot. Laske jonon $(\bar{x}^{(m)})_{m=1}^{\infty}$ raja-arvo.

4. Suppeneeko vektorijono eli pistejono $(\bar{y}^{(k)})_{k=1}^{\infty}$, kun

$$\bar{y}^{(k)} = \Big(\frac{\sin(\pi k/8)}{k^2}, \frac{k}{1+k^2}, \frac{(-1)^k}{k}, \frac{2+k}{k}\Big) \in \mathbb{R}^4 \quad ?$$

Jos suppenee, mikä on jonon raja–arvo?

5. Tarkastellaan seuraavia tason osajoukkoja:

$$A = \{(x,y) \in \mathbb{R}^2 \,|\, y > x, x \in \mathbb{R}\}\ ,\ B = \{(x,y) \in \mathbb{R}^2 \,|\, x > 0, y \le x\},$$
$$C = \{(x,y) \in \mathbb{R}^2 \,|\, y = 2, x \in \mathbb{R}\}$$

Ovatko nämä avoimia tai suljettuja?

6. Onko funktiolla $f : \mathbb{R}^2 \setminus \{\bar{0}\} \to \mathbb{R}$,

$$f(x,y) = \frac{x^2y^2 + 2x^2y}{x^4 + y^2}\ ,$$

raja-arvoa origossa?

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1. Draw figures presenting the following subsets of $\mathbb{R}^3$:
a) $A := B((0,3,3),3)$,

b) $B := \Big\{(x,y,z) \mid 0 \le x \le 2,\ 1 \le y \le 4,\ 1 \le z \le 4, \Big\}$,
c) the set $B \cap B_1$ with
$B_1 := \Big\{(x,y,z) \mid z < 5 - y\Big\}$

2. Let $\bar{a} = (2,0,2) \in \mathbb{R}^3$, and let $\bar{y} \in \mathbb{R}^3$ be a vector with $|\bar{y}| = 1$. Find
a) the maximum length,
b) the minimum length,
for the vector $\bar{a} + \bar{y}$. Find a vector $\bar{y} \in \mathbb{R}^3$, $|\bar{y}| = 1$, which is perpendicular (orthogonal) to $\bar{a}$.

3. Write the coordinate (or component) sequences for the sequence $(\bar{x}^{(m)})_{m=1}^{\infty} \subset \mathbb{R}^3$, where

$$\begin{aligned} \text{a)}\ \ \bar{x}^{(m)} &= (e^{-m^2+m}, 2 + m^{-2}, me^{-m}) \\ \text{b)}\ \ \bar{x}^{(m)} &= (e^{-m^2+m}, 2 + m^{-2}, me^{-m}) + (1, e^{-m}, 2) \\ \text{c)}\ \ \bar{x}^{(m)} &= (m+2)\big(e^{-m^2+m}, 2 + m^{-2}, me^{-m}\big). \end{aligned}$$

Calculate the limit of the sequence $(\bar{x}^{(m)})_{m=1}^{\infty}$.

4. Does the vector sequence $(\bar{y}^{(k)})_{k=1}^{\infty}$ converge, when

$$\bar{y}^{(k)} = \Big(\frac{\sin(\pi k/8)}{k^2}, \frac{k}{1+k^2}, \frac{(-1)^k}{k}, \frac{2+k}{k}\Big) \in \mathbb{R}^4 \quad ?$$

If it does, what is the limit value?

5. Consider the following subsets of the plane:

$$A = \{(x,y) \in \mathbb{R}^2 \,|\, y > x, x \in \mathbb{R}\}\ ,\ B = \{(x,y) \in \mathbb{R}^2 \,|\, x > 0, y \le x\},$$
$$C = \{(x,y) \in \mathbb{R}^2 \,|\, y = 2, x \in \mathbb{R}\}$$

Are they open or closed?

6. Does the function $f : \mathbb{R}^2 \setminus \{\bar{0}\} \to \mathbb{R}$,

$$f(x,y) = \frac{x^2y^2 + 2x^2y}{x^4 + y^2}\ ,$$

have a limit at the origin?