14. Summary

The main material of each of the Chapters is listed below:

Chapter 1:
- Physical interpretation of wave functions
- Basic mathematical background for the course

Chapter 2:
- Hilbert spaces, orthogonal subspaces
- $\oplus$ = Direct sums
- $\otimes$ = Tensor products

$\Rightarrow$ Application: * Spin-s particle wave functions
* N-particle states

Chapter 3:
- Bounded operators = $\mathcal{B}(\mathcal{H})$
- Adjoint in $\mathcal{B}(\mathcal{H})$
- Unitary operators
- Observables, states and C^*-algebraic formulation of QM

Chapter 4:
- Strongly continuous unitary semigroups = quantum dynamics
- Uniform, strong & weak convergence

Chapter 5:
- Unbounded operators: closure & adjoint
- self-adjoint & essentially self-adjoint operators
- Stone's theorem: infinitesimal generator of strongly continuous unitary semigroups & its domain
- Multiplication operators
- Spectral decomposition: spectrum, projection valued measures, operator calculus (Thrm 5.24), eigenvalues

Chapter 6: - Free Schrödinger evolution on $\mathbb{R}^d$
- Schwartz test function spaces S_d and Fourier transform on S_d
- Fourier transform on $L^2(\mathbb{R}^d)$ as a unitary operator
- Differential operators on $L^2(\mathbb{R}^d)$
- Solution of the free evolution using an explicit integral kernel $K(x,y;t)$.

Chapters 7 & 8: - Basics of distribution theory
- Wigner function $W[\psi]$ & "QM phase space"
- Wigner transform $W_{\psi(t)}$
  * Theorem 3.7: Easy transport equation if $\psi(t) = e^{-itH_0}\psi_0$.

Chapter 9: Self-adjoint extensions of symmetric operators
- ... can be done using the Cayley-transform

=> Classification result based on deficiency spaces & indices in Threorem 9.6.
- Boundary conditions on $L^2([0,1])$

Chapter 10: - Relatively bounded operators
- Kato-Rellich theorem
  => $H := H_0 + V$ is self-adjoint on $D(H_0)$ if $d \leq 3$ and $V \in L^\infty + L^2$.
- If V is bounded from below, then $H_0 + V$ can be defined "in the sense of distributions"

Chapter 11: - Examples of various potentials: 1D step potentials, Harmonic oscillators, Hydrogen atom
- Kato's theorem & Molecular Hamiltonians
- Physical systems requiring different H_0:
  * Magnetic fields
  * Relativistic particles

Chapter 12: - "Variable-N" particles systems
& Fock space $\mathcal{F}^{(0)}$
- "N-body potentials"
- Totally antisymmetric subspace $\mathcal{F}^{(-)}$ and fermions
- Totally symmetric subspace $\mathcal{F}^{(+)}$ and bosons
- 2nd quantization and tensor products of operators
- Creation & annihilation operators:
  - *12.4: on $\mathcal{F}^{(0)}$
  - *12.4.1: on $\mathcal{F}^{(-)}$ belong to $\mathcal{B}(\mathcal{F}^{(-)})$
  - *12.4.2: on $\mathcal{F}^{(+)}$ are unbounded but generate a self-adjoint field operator $\Phi(f)$ => Weyl algebra for $W(f) = e^{-i\Phi(f)}$

Chapter 13: - Fermionic lattice systems
= $\mathcal{F}^{(-)}$ with one particle space $\ell_2(\mathbb{Z}^d)$
- One- and two-body interactions B_1 and B_2 given by a_-^* and a_-.

Please return your exam projects to Jani Lukkarinen, preferably by e-mail in a PDF attachment. The oral exam will consist of 5 questions about the topics listed in the above summary.

Good luck!