11. Examples and applications

11.1. 1D step-potentials

* Suppose $V \in C^{(1)}(\mathbb{R}) \cap L^\infty(\mathbb{R})$ is monotone decreasing. Then a classical particle with mass $= 1$ moving under the influence of the potential V has a trajectory x_t which satisfies $\ddot{x}_t = -V'(x_t) \geq 0$. Hence, its velocity $\dot{x}_t$ can only increase. In particular, if $v_0 := \dot{x}_0 > 0$, then $x_t = x_0 + \int_0^t ds\, \dot{x}_s \geq x_0 + v_0 t$ for all $t \geq 0$. This implies that if the particle starts by moving to the right, it can never be reflected by the potential V: we have $x_t \to +\infty$ as $t \to +\infty$.

The situation is very different for quantum particles: not only can they be reflected by such potentials, the reflection can even become near certain.

This phenomena is studied from various angles in the paper (link on webpage and copy as an appendix here): Sec. 2-6. in

P. L. Garrido, S. Goldstein, J. Lukkarinen, R. Tumulka: Am. J. Phys. 79 (2011) 1218-1231
Preprint: arxiv.org/abs/0808.0610.

* In Sec. 7-9, with mathematical details given in the Appendices, we also show how to rigorously connect the "Gamow eigenvalues" $z \in \mathbb{C}$ and the associated 'eigenvectors' $\psi \notin L^2(\mathbb{R})$ satisfying

$$z\psi(x) = -\tfrac{1}{2}\psi''(x) + V(x)\psi(x), \quad x \in \mathbb{R},$$

to the "metastable" escape of particles from a potential plateau $V =$  .