Department of Mathematics and Statistics
Metric Geometry
Exercise 9
20.11.2013

Return by **Tuesday, November 19**.

1. Prove that the κ-cone $X = C_\kappa Y$ over a metric space Y is complete if and only if Y is complete.

2. Let Y be a metric space, $\bar{Y}$ its completion, and $\kappa \in \mathbb{R}$. Prove that $C_\kappa \bar{Y}$ and $\overline{C_\kappa Y}$, the completion of $C_\kappa Y$, are isometric.

3. Suppose that the κ-cone $X = C_\kappa Y$ over a metric space Y is a CAT(κ)-space. Prove that for each pair of points $y_1, y_2 \in Y$, with $d(y_1, y_2) < \pi$, there exists a unique geodesic segment in Y joining y_1 and y_2.

4. Let Y be a length space and $\kappa \in \mathbb{R}$. Show that $C_\kappa Y$ is a length space.

5. Let (X, d) be a metric space of curvature $\leq \kappa$. For each $n \in \mathbb{N}$, we define a metric d_n by setting
$$d_n(x, y) = n\, d(x, y).$$
Prove that the tangent cone $C_0 S_p(X)$ at $p \in X$ (and its completion) is a 4-point limit of the sequence (X, d_n).