

Department of Mathematics and Statistics
Metric Geometry
Exercise 3
2.10.2013

Return by **Tuesday, October 1.**

1. Let (\mathbb{R}^2, d) be a metric space, where

$$d((x_1, y_1), (x_2, y_2)) = |x_1 - x_2| + \sqrt{|y_1 - y_2|}.$$

Find the generalized inner metric d_s associated to d . What is the topology \mathcal{T}_{d_s} determined by d_s ?

2. Let X be a length space and $x, y \in X$, $x \neq y$. Prove that

$$\text{dist}(x, B(y, r)) = |x - y| - r$$

if $r < |x - y|$.

3. Prove that the completion of a length space is a length space.
4. Construct a complete length space which is not a geodesic space.
5. Construct a locally compact geodesic space whose completion is neither geodesic nor locally compact.