Department of Mathematics and Statistics
Metric Geometry
Exercise 10
4.12.2013

Return by **Tuesday, December 3**.

Given $a, b, c > 0$, a tripod $T = T(a, b, c)$ is a metric tree consisting of three edges of lengths a, b, c meeting at a common vertex (of valence 3). For convenience, we extend the notion of a tripod to the cases where a, b, and c are allowed to be zero.

1. Let X be a metric space and $x, y, z \in X$. Prove that there exists a tripod T and an isometric embedding $f\colon \{x, y, z\} \to T$ (into) such that $(y|z)_x = \ell(a)$, the length of the edge with $f(x)$ as an endpoint.

2. Suppose that X is (Gromov) δ-hyperbolic. Prove that
$$|w - y| + |x - z| \leq \max\{|x - y| + |w - z|, |x - w| + |y - z|\} + 2\delta$$
for every $x, y, z, w \in X$.

3. Suppose that X is a 0-hyperbolic geodesic metric space. Prove that any pair of points can be joined by a unique geodesic segment.

4. It is known that every geodesic triangle in $\mathbb{H}^2$ has an area at most π (follows from the Gauss-Bonnet theorem). Use this knowledge to verify that M^2_κ is δ-hyperbolic for some $\delta \geq 0$ if $\kappa < 0$.

5. Let X be a δ-hyperbolic geodesic space. Prove that X satisfies the Rips condition with 4δ.

6. Detect and correct all misprints and incorrect statements above (if any).