Malliteoria Harjoitus 4

1. Where in the proof of Theorem 5.13 the atomic formula  $\top$  was needed?

2. Let  $L = \{E\}, \ \#E = 2$ , and  $T_g$  consist of the following two sentences:

$$\forall v_0 \neg E(v_0, v_0)$$
$$\forall v_0 \forall v_1 (E(v_0, v_1) \rightarrow E(v_1, v_0)).$$

Show that T has AP, JEP and is closed under unions.

3. Let L and  $T_g$  be as above. Find a theory T such that the models of T are exactly the existentially closed models of  $T_q$ .

4. Let T be as in Example 5.19. Show that T has AP, JEP and is closed under unions.

5. Let T and  $\phi$  be as in Example 5.19. Show that if  $\mathcal{A}$  is an existentially closed model of T and  $a \in \mathcal{A}$ , then  $\mathcal{A} \models \phi(a)$  iff  $(c_i^{\mathcal{A}}, a) \in Q^{\mathcal{A}}$  for some  $i < \omega$ .

6. Let T and  $\phi$  be as in Example 5.19. Show that there is no quantifier free formula  $\psi(v_0)$  such that for all existentially closed models  $\mathcal{A}$  of T,  $\mathcal{A} \models \forall v_0(\phi \leftrightarrow \psi)$ .