Malliteoria

Harjoitus 2

1. Exercise 2.9.

In the next three exercises, $U \subseteq \mathcal{P}(\omega)$ is such an ultrafilter that for all $n \in \omega$, $\{x \in \omega |\ x \geq n\} \in U$.

2. Suppose that for all $i \in \omega$, $\mathcal{A}_i$ is a finite structure. Show that $|\Pi_{i \in \omega} \mathcal{A}_i / U| \neq \omega$.

3. For all $i \in \omega$, let $\mathcal{A}_i = (\mathbf{Q}, <)$. Show that there is $f : (\mathbf{R}, <) \to \Pi_{i \in \omega} \mathcal{A}_i / U$ such that for all $x, y \in \mathbf{R}$, $x < y$ iff $f(x) < f(y)$.

4. Suppose that for all $n \in \omega$, $\mathcal{A}_n = (n+1, <)$. Show that $\Pi_{i \in \omega} \mathcal{A}_i / U$ is not a well-ordering.

5. Show using compactness theorem, that there are no $\{f\}$-theory T, $\# f = 1$, such that $\mathcal{A} = (\mathcal{A}, f) \models T$ iff for all $x \in \mathcal{A}$ there is $n \in \mathbb{N} - \{0\}$ such that $f^n(x) = x$, where, as in the first exercise, $f^0(x) = x$ and $f^{n+1}(x) = f(f^n(x))$.