Malliteoria

Harjoitus 5

1. Let $L = \{<\}$, $<$ is a 2-ary predicate symbols, and let $T_{lo}$ ($lo$ for linear ordering) consist of the following sentences:

$\forall v_0 \forall v_1 \forall v_2 ((v_0 < v_1 \wedge v_1 < v_2) \rightarrow v_0 < v_2)$

$\forall v_0 \forall v_1 (v_0 < v_1 \rightarrow \neg v_1 < v_0)$

$\forall v_0 \forall v_1 (v_0 < v_1 \vee v_0 = v_1 \vee v_1 < v_0)$.

Show that $T_{lo}$ has AP, JEP and is closed under unions.

2. Let $T_{lo}$ be as above. Find a theory $T$ so that the models of $T$ are exactly the existentially closed models of $T_{lo}$.

3. Exercise 6.6.

4. Assume that $T$ is an $L$-theory and $|L_{\omega\omega}| = \omega$. Show that if for all countable $\mathcal{A}, \mathcal{B} \models T$, $\mathcal{A} \subseteq \mathcal{B}$ implies $\mathcal{A} \preceq \mathcal{B}$, then $T$ is model complete.

5. Let $L = \{P_0, P_1, R\}$ be a vocabulary such that $P_0$ and $P_1$ are unary relation symbols and $R$ is a binary relation symbol. Let $\mathcal{A}$ be an $L$-structure such that $dom(\mathcal{A}) = \omega$, $P_0^{\mathcal{A}} = \{0\}$, $P_1^{\mathcal{A}} = \{1\}$ and $R^{\mathcal{A}} = \{(n, 0) |\ 1 < n < \omega \text{ even}\} \cup \{(n, 1) |\ 1 < n < \omega \text{ odd}\}$. Show that $Th(\mathcal{A})$ is model complete but does not have the elimination of quantifiers.

6. Find a theory $T$ such that it is model complete but not complete.