

To be returned on Wednesday, September 25, 10.15am at latest

1. Find all characters of irreducible representations of the cyclic group  $C_n$  (which consists of powers of  $e^{2\pi i/n}$ ).
2. Find all conjugacy classes in the dihedral group  $D_n$ . (See the problems 1/2 and 4/2.)
3. Let  $G$  be the group of all invertible real  $n \times n$  matrices  $A$  such that  $A_{ij} = 0$  for  $i > j$  and  $A_{ii} = 1$ . Define a left invariant integration on  $G$ . Hint: Try first  $n = 2, 3$ . You just need to use the standard invariance properties of integration in an Euclidean space  $\mathbb{R}^k$ . Is the integral also right invariant?
4. On the bases of the exercise 2 we know the number of inequivalent irreducible representations of  $D_n$ . Find first all 1-dimensional representations. It turns out that the rest of the irreps are 2-dimensional; try to construct some of them. Compare with the number of conjugacy classes in  $D_n$ ! It turns out to be useful to separate the cases  $n$  even and  $n$  odd.