Introduction to probability with MATLAB, Spring 2014
University of Helsinki, Department of Mathematics and Statistics
Exercise set 9

1. Compute the exact distribution (i.e. point probabilities) of $S = X + Y$, where X is the result from a regular 6-sided die, and Y is the result from a 3-sided die (it generates integers $1, 2, 3$ equiprobably). Hint: There are 6×3 elementary events. Find which elementary events give which sums. Draw the point probabilities and notice the trapezoidal shape, similar to the arrival time of Mr. N's first bus (exercise 7:12).

2. The following program solves the exact distribution of $S = X + Y$, where X and Y are two throws of regular 6-sided dice.

```
Ps = zeros(1,12);
for x=1:6
  for y=1:6
     Pxy = 1/36;
     s = x+y;
     Ps(s) = Ps(s) + Pxy;
   end
end
```

The probability of each elementary event $(X = x, Y = y)$ is $1/36$, so the program lists every such event and adds that probability to the probability of $S = x + y$. The probabilities are collected as a vector such that `Ps(s)` $= P(S = s)$. Note that this is full enumeration of the $6^2 = 36$ elementary events, not a random simulation, so we get the exact probablities. Modify the program to solve the distribution of $S = X + Y + Z$ where X, Y, Z are **three** throws of 6-sided dice. Plot the point probabilities with `bar`.

3. Modify the program to solve the distribution of $S = X + Y + Z$, where X, Y, Z are throws of a biased coin: with probability $p = 0.3$ it results heads (one) and otherwise tails (zero). Do not randomize, use the exact probabilities. Note that different values of (x, y) have different probabilities $P(X = x, Y = y)$, so you must compute that probability inside the loop, it is not a constant like $1/36$.

4. We throw a regular die until we get six; the count of throws is the random variable X. After that, we throw the die again until we get six; the count of these throws is Y. Obviously X and Y are independently geometrically distributed with parameter $p = 1/6$.

Let now $S = X + Y$, i.e. the number of throws needed until we have collected two sixes. Now the number of elementary events (the cardinality of the sample space) is infinite, so obviously we cannot enumerate all elementary events. Modify the program to compute $P(S = s)$ for $s = 1, 2, \ldots, 50$. Hint: For a given s there are only a finite number of possible values of (x, y).

Plot the point probabilities with `bar`. Note that the distribution is not geometric. It is called the *negative binomial distribution*.

5. Why cannot we as easily use a similar program to find the exact distribution of $S = X+Y$ where X and Y have the uniform distribution over $(0, 1)$?

6. Use a random simulation and a histogram to approximate the distribution of $S = X_1 + \ldots + X_n$, where n is a fixed small integer, and $X_1, \ldots, X_n$ have independent exponential distributions with mean $\mu = 10$. Do this for $n = 2$, 3, 10 and 20. Hint: use `exprnd(10,n,N)` to generate n rows and N columns of such numbers. Use `sum` to take the column sums. Take $N = 10^6$ to get a fairly large sample, and use 100 bars in the histogram.

7. A hollow cylinder has inner diameter $i = 98$ mm, and thickness $h = 2$ mm, thus the outer diameter is $o = 100$ mm. We have the bright idea to measure the two diameters and compute the difference to obtain the thickness. We get measurements $I = i + e_1$ and $O = o + e_2$, where the measurement errors e_1, e_2 have independent normal distributions with mean 0 and standard deviation 1 mm.

(a) What is the distribution of our measured thickness $T = O - I$?

(b) What is the distribution of its error $T - t$, where t is the correct thickness $t = o - i$?

(c) What is the probability that the measured thickness is negative?

8. Two persons A and B have agreed to meet at the railway station. A takes a bus from Kontula and B takes a bus from Lauttasaari to the railway station. They depart simultaneously. The driving time for A's bus has distribution $N(25, 8^2)$, and the driving time for B's bus has distribution $N(30, 6^2)$. Let X be difference of their arrival times to the railway station (positive: A arrives first; negative: B arrives first).

(a) What is the distribution of X?

(b) What is the probability that B arrives first?

(c) What is the probability that A has to wait more than 5 minutes?

(d) What is the probability that B has to wait more than 5 minutes?

(e) What is the probability that one of them has to wait more than 5 minutes?

9. Study empirically the additivity of normal distribution. Let X and Y have independent normal distributions, possibly with different parameters. Take large random samples of X and Y and look at the histogram of $X + Y$. Observe how neatly the sum always retains the same normal distribution shape, regardless of what the parameters are.