Introduction to probability with MATLAB, Spring 2014
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Exercise set 7

1. (Independence revisited) Your company is in the process of designing a passenger airplane. All the flight control surfaces (those flaps that are turned in order to turn the aircraft up, down, right or left) are hydraulically powered. For safety you are planning to have **three** physically separate hydraulic systems, such that even if two of them fail, one functional system is enough to control the airplane.

Based on some statistics and engineering calculations, you are informed that during any given flight, a hydraulic system will fail with probability $p = 0.001$. Calculate the probability that all three systems fail (leading to a disaster), during a given flight.

2. Previous continued. It turns out that the $p = 0.001$ in the previous problem accounted only for independent failure of a hydraulic system. Independent of that, we have found out that during any given flight, the aircraft may experience another failure (with probability $q = 10^{-5}$), such that it will cause all hydraulic systems to fail simultaneously. Calculate the probabilities of the following events:

- A_i $(i = 1, 2, 3)$, denoting the event that system i fails (for one reason or another)
- $A = A_1 \cap A_2 \cap A_3$, denoting the event that all systems fail.

Based on the calculation, explain whether the events A_i are independent (from each other).

Background. A real-life example would be United Airlines flight 232 in 1989; a rear engine failed in such a way that shrapnels from the engine severed all three hydraulic systems. This exercise is loosely based on UA 232 but is not an exact reproduction.

3. Previous continued. The $P(A)$ we calculated seems unacceptably high, given that we are going to manufacture hundreds of aircraft of this type and there are going to be hundreds of thousands of flights on them. Try to think some ways of reducing considerably the probability that all flight controls fail (leading to disaster). In particular, consider the alternatives (a) let's add a fourth hydraulic system of the same kind, (b) let's add a completely different mechanism for controlling the flight control surfaces.

4. A train is supposed to arrive at 12:20. The actual arrival time differs from the schedule by a random amount X, which has an uniform distribution over the interval $(-3, 7)$. (Negative amount means that the train arrives before scheduled time.) What is the probability that

(a) the train arrives in time (= at scheduled time or earlier)?

(b) more than 5 minutes late?

(c) between 12:22 and 12:23?

5. Mr. K is waiting for a tram. The tram goes exactly according to a schedule, with exactly 10-minute intervals. However, Mr. K arrives at the tram stop at a random time.

(a) What is the distribution of the waiting time?

(b) What is the probability that the tram will arrive within two minutes of Mr. K's arrival?

6. Previous continued. After the (random) arrival, Mr. K has now waited 5 minutes. What is now the probability that the tram will arrive within the next two minutes? (Hint: Conditional probability. Consider Mr. K's arrival time within the 10-minute interval, and think what arrival times are now ruled out because the tram *did not arrive* within 5 minutes of his arrival.) Also, calculate the same when Mr. K has waited 9 minutes.

7. A real number X is chosen at random from the unit interval $(0, 1)$, i.e. it is drawn from the uniform distribution over that interval. X is represented as a decimal number (possibly infinitely long). What is the probability that

(a) the first digit (after decimal point) is 3?

(b) the second digit is 3?

(c) both of them are threes?

8. G&S chapter 2.2, exercise 1 (page 71).

9. G&S chapter 2.2, exercise 5 (page 71).

10. G&S chapter 2.2, exercise 17 (page 72).

11. Mr. N arrives at a bus stop at time 0. The arrival time of the bus is A, which has uniform distribution between 1 and 5 minutes. Mr. N will then ride the bus to another stop where he will try to change to another bus. The first bus takes X minutes, uniformly distributed between 10 and 20, to reach the second stop. The second bus will arrive there at time Y, which is uniformly distributed between 17 and 23. The change will be successful if and only if the first bus arrives before the second bus.

Try to estimate *experimentally* the probability that Mr. N makes it into the second bus. Simulate his travel 10 000 times, each time taking the three values A, X and Y from the uniform distribution (`unifrnd`), and each time checking whether the change is successful. Compute the relative frequency.

12. Previous continued. Draw a histogram (`hist`) of the time when the first bus arrives on the second stop. Does it seem to have a uniform distribution? What about the transfer marginal Z, defined as the difference of the arrival times of the buses (positive = bus 1 arrives first, negative = bus 2 arrives first). What do you know about the distribution of Z? What is it expected value? What are its possible values?