

Introduction to probability with MATLAB, Spring 2014
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Exercise set 6

1. Alice and Bob play the following game: Each player throws one die on his or her turn, and wins if the result is 6. Alice plays first (then Bob, Alice, Bob, etc.). Let the turns be indexed $1, 2, 3, \dots$, so Alice plays on the odd turns and Bob plays on the even turns. Let N be the index of the turn on which the game ends. What is the distribution of N ?
2. Previous problem continued. Alice wins if the game ends on an odd-numbered turn. What is the probability that Alice wins? (Hint: Geometric series.)
3. If Alice wins, she gains 100 euros, otherwise she loses x euros. What is her expected gain (losing is considered a negative gain)? How large may x be so that her expected gain is positive?
4. A die is tossed repeatedly until the result is six. X is the number of tosses. Find the following quantities:
 - (a) the *mode* of X (i.e. the value that has the largest probability)
 - (b) a *median* of X
 - (c) the expected value of X
5. A die is tossed repeatedly until we have obtained *ten* sixes. What is the expected value of the number of tosses?
6. A coin is tossed, and a square is then drawn. If the coin is heads, the square has side $X = 1$. If the coin is tails, the square has side $X = 10$. The area of the square is of course $A = X^2$. Find $E(X)$ and $E(A)$. Is the latter equal to $E(X)^2$? Why / why not?
7. G&S, Chapter 4.1, exercise 23 (page 153).
8. G&S, Chapter 6.1, exercise 13 (page 248).
9. G&S, Chapter 6.1, exercise 19 (page 249).