

Introduction to probability with MATLAB, Spring 2014
University of Helsinki, Department of Mathematics and Statistics
Exercise set 5

1. (Coupon collecting) Boxes of breakfast cereal contain collectible cards with fancy pictures. There are five different cards with different pictures, numbered 1, 2, 3, 4 and 5. Each box contains randomly one of the cards. Peter's mother buys one box every week.

Let X_i denote the number of boxes Peter has to open to get i th *new* card (with a picture he does not already have), *after* he already has collected $i - 1$ cards.

Obviously $X_1 = 1$ since Peter gets a card from the first box, and it is certainly new. What is the distribution of the random variable X_2 ? What is its expected value?

2. Previous problem continued. What are the distributions and expected values of X_3 , X_4 and X_5 ?

3. Previous problem continued. What is the expected value of $Y = X_1 + X_2 + X_3 + X_4 + X_5$, the total number of boxes that Peter has to open until he has the full set of five cards? Is it easy to deduce the distribution of Y , that is, the probabilities $P(Y = k)$, for $k = 1, 2, \dots$?

4. Previous problem continued. Write a Matlab program that simulates the random variable Y , i.e. generates a value $Y = k$ according to the distribution. (Hint: Geometric distribution can be simulated with the auxiliary function `coinuntil` which you can find from the course page.)

Simulate 1000 times the experiment "Peter opens boxes until he has 5 different cards" and draw a frequency plot (`freqplot`) of the values of Y you got. Compute also the arithmetic mean (average) of the values that you got (hint: `mean`). Compare to problem 3.

5. G&S, Chapter 5.1, exercise 1 (page 197).

6. G&S, Chapter 5.1, exercise 7 (page 197).

7. G&S, Chapter 5.1, exercise 28 (page 201).

8. In a classroom there are 3 students, named Alice, Bob and Charlie, of heights 160, 170, and 180 cm. One student is chosen at random, and X is the height of this student. What is the distribution of X ? What is its expected value?

9. Previous problem continued. *Two* students are chosen at random (without replacement) from the three students. Let X be the height of the first student chosen, and Y the height of the second one. What is $E(Y)$?

10. Previous problem continued. Let $S = X + Y$. What is the distribution of S (i.e. what are the possible values of S and their probabilities)? What is $E(S)$?

11. Previous problem continued. What is the conditional distribution of Y , given X ? (That is, what are the conditional probabilities $P(Y = y \mid X = x)$, for all possible values of x and y .)

12. A 6-sided die is tossed three times. Let M be the *maximum* (i.e. biggest) of the three results. What is the distribution of M ? What is the expected value of M ? Hint: Try first to deduce the probability $P(M \leq m)$, where m is an integer, by thinking about a Bernoulli trial where "a die thrown was at most m " is "success". Then think about the events $M \leq m$ and $M \leq m - 1$, and try to deduce the probability $P(M = m)$.

13. Simulate the experiment of the previous problem many times. Plot a frequency plot of M and compute the average.