Introduction to probability with MATLAB, Spring 2014
University of Helsinki, Department of Mathematics and Statistics
Exercise set 4

1. A factory produces widgets. Each day the widget-producing machine is either in order (with probability 0.99) or out of order (with probability 0.01). Days are independent from each other.

If the machine is in order, each widget produced is GOOD with probability 0.99, and otherwise BAD. Widgets are independent.

If the machine is out of order, each widget produced is GOOD with probability 0.4, and otherwise BAD. Widgets are independent.

You do not know the status of the machine today, but you examine one widget. What is the probability that the widget is good?

Hint: Consider the two events A = "machine is in order, and widget is good", and B = "machine is out of order, and widget is good". Are these two events disjoint? What event does their union represent? This is also know as the "law of total probability".

2. Problem 1 continued. You do not know the status of the machine today, but you have examined one widget, and it is BAD. What is now the probability that the machine is in order?

3. Problem 1 continued. You do not know the machine status, but you examine 10 widgets produced today. What is the probability that exactly 3 of them are BAD? (Hint: Total probability again.)

4. Problem 1 continued. You do not know the machine status, but you have examine 10 widgets produced today, and found that 3 of them are BAD. What is now the probability that the machine is in order?

5. Mr. K is going to mail two books to his friend, of values 10 EUR. and 20 EUR. Any packet that he sends will disappear with probability 0.1. Mr. K ponders whether he should send the books separately or in one packet. Explain what he should do, if he wishes to

a) minimize the expected value of books lost,

b) maximize the probability that the friend received both books,

c) maximize the probability that the friend received at least one book.

6. A fair coin is tossed 100 times. Let X be the random variable that denotes the number of "heads". (We then know that X has a binomial distribution.)

a) Calculate the expected value $\mu = E(X)$.

b) What is the probability $P(X = k)$, for any integer k?

c) What is the probability $P(X = \mu)$?

d) What is the probability $P(\mu - 3 \leq X \leq \mu + 3)$?

7. Previous problem continued. Compute the probability $P(X = k)$ for all possible values $k = 0, \ldots, 100$. Plot the values (hint: `bar`). Is the plot familiar? Explain in words what is a typical result of the "coin tossed 100 times" experiment.

8. A 6-sided die is tossed twice. Let X be the sum of the two results. What is the expected value of X? Try to find out, at least approximately, the distribution of X (i.e. the probabilities of all possible events $X = k$), either by calculations or by experiment.

9. Same as previous problem, but the die is tossed ten times.