Introduction to probability with MATLAB, Spring 2014
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Exercise set 3

1. A box contains 15 balls (5 white and 10 black). Three balls are drawn at random, *without replacement*. Calculate the probability that we get (a) exactly 1 white ball, (b) no white balls, (c) only white balls.

2. Previous problem continued. Given that we have 5 white balls out of 15 total, and we are interested in the number of white balls in a sample, does it matter what colors the other 10 balls are?

Suppose those 10 other balls are: 2 black, 2 red, 1 purple, 4 green and 1 yellow. What are the probabilities for events (a)–(c)?

3. A certain population consists of $N = 1000$ people. 500 of them support party A; 300 support party B; and 200 support party C. A sample of three persons is drawn at random, without replacement. What is the probability that all persons in the sample support party A? (Compare to the previous problems.)

4. Same as previous problem, but the sample is drawn *with replacement*. What is the probability? Explain the difference of these samples.

5. MATLAB exercise, continuing the previous problems. Let the people in the population be labeled with numbers $1, \ldots, 1000$ such that persons labeled $1, \ldots, 500$ support party A, persons $501, \ldots, 800$ support party B, and persons $801, \ldots, 1000$ support party C.

Draw a sample of three people, *without replacement*, using `randperm(1000,3)`. Repeat a few times. Try to construct an expression that gets the value 1 if a sample contains only supporters of A, and 0 otherwise.

6. Continued. Write a `for` loop that draws 10000 three-person samples from the same population, and counts how many times it happened that “sample contains only supporters of A”. Compute the relative frequency (i.e. count of occurrences divided by the total count of trials). (Use `HELP FOR` to learn the syntax.) Compare the experimental result to what you calculated problem 3.

7. A class of $4n$ children contains $2n$ boys and $2n$ girls. A group of $2n$ children is chosen at random. Write down the probability that the group contains an equal number of boys and girls. Note that this is a function of n. Experiment with a few values of n, for example $n = 10, 20, 30, 100$. (You can calculate the binomial coefficients in MATLAB with `nchoosek`.) Is the result surprising?

8. Experiment with the sampling described in the previous problem by drawing random samples with MATLAB (the method would be similar to problem 6).

9. One card is drawn from an ordinary deck of 52 cards. Note: in such a deck, there are four suits of 13 cards; two suits black (spades and clubs), and two suits are red (hearts and diamonds).

Consider the following four events pairwise (there are 6 pairs of events, why?). For each pair of events, explain whether the two events are *independent* or not; and also, whether they are *disjoint* or not. (Disjoint events are such that they cannot both happen.)

A = “The card is a spade”

B = "The card is a club"

C = "The card is black"

D = "The card is an ace"

10. A pin is tossed on a table n times. Each time the tip lands "up" with probability $p = 0.3$ and "down" with probability $q = 0.7$. The results of the tosses are independent, so the situation can be modelled as a Bernoulli experiment. Let A_k denote the event that we get exactly k "up" results.

(a) If $n = 2$, compute the probabilities $P(A_0)$, $P(A_1)$ and $P(A_2)$.

(b) If $n = 10$, compute the probability $P(A_3)$.

(c) Write down a general expression for $P(A_k)$ if $k = 0.3n$. You can asume that n is an integer multiple of 10, so that k is guaranteed to be an integer. How does this probability seem to behave as n grows? How do you explain the result?

11. Continued. Repeat 1000 times the experiment "the pin is tossed 100 times". (Hint: Use a for loop; you can toss a pin n times with the command `coin(n, 0.3)`, where the pin is thought to be a "biased coin" with 0.3 probability for heads.)

For each of the 1000 experiments, count how many "up" results you got. Collect these counts as a vector of 1000 numbers (hint: start with an empty vector `A=[]`, then on ith experiment, assign `A(i) = something`).

How often did you get exactly $0.3n = 30$ "up" results? Does the relative frequency agree with the result from the previous problem? Draw a frequency plot (`freqplot`) of the "up" counts. Which counts seem to occur relatively often? Do you think it is possible to get 100 "up" results when tossing the pin 100 times?