Introduction to probability with MATLAB, Spring 2014
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Exercise set 2

The MATLAB functions `coin`, `dice` and `freqplot` can be found on the page `http://www.helsinki.fi/~kohonen/intropro b/` (link is also on the course webpage). You must save them to your local MATLAB folder so that MATLAB can find them.

1. A six-sided die is tossed once. What is the probability that the result is "big" (defined here as "five or six")? Calculate the exact result.

Now try to evaluate the probability of the same event **experimentally**. Throw a die 10 times. You can simulate die-tossing with the MATLAB function `dice`. What was the **relative frequency** of "big" results out of the ten tosses? What if you toss another 10 times?

2. Previous problem continued. Experiment with toss sequences of different lengths, for example 10^k tosses where $k = 2, 4, 6, 8$. How many correct decimal places do you seem to get? (Use the MATLAB command `format long` to see more decimals.)

Can you guess how the number of correct decimals seems to depend on k? You don't have to prove the guess (we'll come to that later during the course). How many times would you have to toss a die so that you would be reasonably sure of obtaining ten correct decimals? How long would that take?

3. Three different coins (bronze-colored, silver-colored, and gold-colored) are tossed. Let us denote the results with three integers variables B, S and G, with the value 1 indicating heads and 0 tails. Write down the following events as logical expressions, and calculate their probabilities.

(a) Both the bronze and the silver coin are heads.

(b) All three coins are heads.

(c) At least one of the coins is heads.

4. Previous problem continued. Try to evaluate experimentally the probabilities of the three events a–c by tossing each coin 1000 times (use the function `coin`). Save the results of the tosses in three 1000-element vectors `R`, `H` and `K`, and compute the relative frequency of each event in the experiment.

5. Toss two dice 1000 times each, and each time calculate the sum of the two dice (an integer between 2 and 12). Draw a frequency plot of the sums (use `freqplot`).

How many times was the sum equal to 2, and how many times was it 7?

After the experiment, do you think that the sum seems to have an equal probability of being any one of the integers from 2 to 12?

6. Suppose A,B,C are three events such that $P(A) = 0.1$, $P(B) = 0.1$ and $P(C) = 0.2$. What can you say about $P(A \cup B \cup C)$? What about $P(A \cap B \cap C)$?

7. Prove the Boole inequality for two events:

$$P(A \cup B) \leq P(A) + P(B).$$

8. Prove the Boole inequality for n events:

$$P\left(\bigcup_{i=1}^{n} A_i\right) \leq \sum_{i=1}^{n} P(A_i).$$

(Use induction.)

9. A four-sided die, having the shape of a tetrahedron, has been loaded unevenly. The outcome of a toss is determined by which of the four corners (labeled A, B, C and D) points up. In a very long experiment we have observed that the frequencies of the four corners are proportional to 3, 4, 5 and 8. What are the probabilities of the four outcomes?

10. From a set of five persons (A, B, C, D and E) a random subset of two persons is selected. Each such subset is equiprobable.

(a) *Enumerate* (list) all possible such subsets, either by hand or in Matlab with `nchoosek('ABCDE',2)`. How many possibilities are there?

(b) Write down a formula for *calculating* the number of such subsets.

(c) What is the probability of each possible subsets?

(d) What is the probability that person A is selected? What about person B?

11. A restaurant menu has 2 diffent soups, 4 different first courses, 3 different main courses, and 3 different desserts.

(a) How many different full meals (soup, first course, main course, and dessert) can you choose? (Apply the combinatorial *rule of product*.)

(b) In each section of the menu (soups, first courses etc.) one of the choices contains nuts. Charles is allergic to nuts, and picks a meal at random out of all possible choices. What is the probability that his meal does *not* contain nuts? (For example, first *count* the number of all non-nuts meals, then calculate the quotient.)

12. From a deck of 52 cards, four cards are drawn. What is the probability that the four cards **(a)** all have the same value **(b)** all have different values? You may do the calculation either by counting ordered sequences of four different cards, or by counting permutations (unordered sets of four cards), whichever seems more suitable.

13. The card "3 of spades" has been removed from a deck of cards. Now, if two cards are drawn from the remaining deck of 51 cards, what is the probability that those two cards are

(a) a pair of threes

(b) a pair of fours

(c) a pair of any value?