Introduction to probability with MATLAB, Spring 2014
University of Helsinki, Department of Mathematics and Statistics
Exercise set 11

1. G&S exercise 9.1.1 (page 338).

2. G&S exercise 9.1.2 (page 338).

3. G&S exercise 9.1.4 (page 338).

4. A single experiment consists of rolling six dice, and we define "success" as the event "all six dice are showing a different result". Calculate the probability of this event. (Hint: Back to basics. Count the favorable outcomes, and count all possible outcomes. Alternatively, you can use the multinomial probability.)

5. Previous continued. The same experiment (of rolling six dice) is performed 3000 times. We are interested in X, the number of successes. What is its exact distribution? Calculate its mean and standard deviation. Then *approximate* that X has the normal distribution with the same mean and standard deviation, and using this approximation, calculate $P(X > 50)$.

6. G&S exercise 9.2.3 (page 354).

7. "Hundred-year flood". We are modelling the spring flood in River Whatsthename. Each year i we record the highest water level (in meters, relative to some arbitrary zero level) during that spring, and call that H_i. It seems that the H_i are independent between years, and follow the standard normal distribution $H_i \sim N(0, 1^2)$.

(a) What is the probability that $H_i \leq 2$ meters, for a given year i?

(b) What is the probability that $H_i \leq 2$ meters for 10 consecutive years $i = 1, 2, \ldots, 10$? (Hint: Independence.)

(c) What is the probability that $H_i \leq 2$ meters for 100 consecutive years?

(d) What is the probability that there *is* a flood higher than 2 meters *at least once during 100 years*?

(e) What is the *expected number* of over 2-meter floods during 100 years? (Hint: What is the expected number of such floods during 1 year? By "expected number" we mean the expected value of the number of such floods.)

8. Previous continued. Let M denote the highest flood during 100 years, that is, $M = \max\{H_1, H_2, \ldots, H_{100}\}$.

(a) What is the probability that $M \leq m$, where m is a given real number? By solving this you have found the cumulative distribution function $F_M(m)$.

(b) Solve the density function $f_M(m)$ by taking the derivative.

(c) Plot the density function by evaluating it at points `m = linspace(0,5)`. (That function gives you a set of equally spaced points from 0 to 5.) Does M seem to have a normal distribution? If not, how is it different?