Introduction to probability with MATLAB, Spring 2014
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Exercise set 10

1. A rectangular plot of land is measured. The true length is $a = 20$ meters and true width $b = 30$ meters, and the true perimeter is $p = 2a + 2b$. However, the measured length is $A = a + X$ and measured width is $B = b + Y$, where X and Y are independent errors. According to the measurements the perimeter is $P = 2A + 2B$. **We do not know the form of the distribution of X and Y**, but we know that $E(X) = E(Y) = 0$ and $D(X) = D(Y) = 1$ meter. Compute

(a) expected value and standard deviation of the measured perimeter P,

(b) expected value and standard deviation of its error $P - p$.

2. (Much harder than previous exercise.) The true area is ab and the measured area is $AB = (a+X)(b+Y)$. Compute the expected value and standard deviation of the measured area.

3. Problem 1 continued. What if we measure all four sides of the rectangle, with four independent measurement errors with mean zero and standard deviation 1, and add the four measurements to obtain the perimeter. Is the standard deviation of the error now bigger or smaller than previously?

4. G&S exercise 8.1.1 (page 312).

5. G&S exercise 8.1.2 (page 312). In Matlab you can use the function `binopdf`. Note that `binopdf` can be applied to a vector of values, for example `0:35` computes the point probabilities for all $k = 0, \ldots, 35$.

6. A regular die is rolled until we have obtained ten sixes. As we know, the number of rolls for getting one six (after the previous one) has a geometric distribution. Let Y denote the number of rolls until we have ten sixes. Calculate $\mu = E(Y)$ and $\sigma = D(Y)$.

7. Previous continued. Using Chebysev's inequality, calculate an upper bound for the probability $P(|Y - \mu| > \epsilon)$, where $\epsilon = 2\sigma$.

8. G&S exercise 8.2.5 (page 321).

9. A biased coin with heads probability $p = 1/3$ is tossed n times. Let Y denote the number of heads, and $X = Y/n$ their relative frequency. Calculate $E(X)$, $V(X)$, and $D(X)$.

10. Previous continued. Let $\epsilon = 0.01$. We are hoping that X would approximate the true probability ($p = 1/3$) within ϵ, in other words, we are hoping that $|X - p| < \epsilon$. The negation of that event is $|X - p| \geq \epsilon$. If this happens we say that the *error is too large*, and we shall use the letter L to denote this "large error" event. Using Chebysev's inequality calculate an upper bound for the probability $P(L)$. Note that this is an expression involving n.

11. Previous continued. How large has n to be so that Chebysev's inequality guarantees that $P(L) < 0.05$?

12. Choose n as in the previous problem. Then toss a simulated coin n times, compute X, and check what was the error $|X - p|$. Was it within the bounds that we hoped (i.e. smaller than ϵ)?